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Welfare implications of personalized pricing in competitive platform markets: The role of network effects

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Welfare implications of personalized pricing in competitive platform markets: The role of network effects*

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Abstract

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1 Introduction

Two primary trends are discernible alongside the growing maturity of complementary technologies in digital markets. First, firms recognize the inherent value in cross-side network benefits and are increasingly adopting the platform business model by extending their product offerings to third-party creators. Second, platforms are progressively utilizing sophisticated tools to track consumers.[1] The implementation of sophisticated data analytics tools in consumer tracking enhances the value of platforms, enabling a deeper understanding of consumer needs, preferences, and willingness to pay. Consequently, we observe digital multi-sided platforms resorting to personalized pricing.[2] This practice enables platforms to extract consumer surplus more effectively (Wagner and Eidenmüller, 2019). Notably, recent studies by Shiller (2020), Dubé and Misra (2023), and Smith et al. (2023) have provided quantitative evidence supporting the effectiveness of such practices. Thus, regulators are keen to understand how such (personalized) pricing schemes affect competition and consumers. Recently, the Federal Trade Commission (FTC) launched an inquiry into the welfare effects of "surveillance pricing."[3] Therefore, studying how these technologies and pricing schemes affect consumers and competition is relevant and can directly inform policymakers.

The extant literature on the effects of competitive personalized pricing on profits and consumers is ambiguous. A vast body of literature posits that personalized pricing is detrimental for firms compared to uniform pricing in oligopoly markets (Thisse and Vives, 1988, Shaffer and Zhang, 1995, Zhang, 2011). However, such claims are challenged by the increased prevalence of personalized prices adopted by (competing) platforms.[4] In

[1] Platforms leverage data collected from personal devices such as smartphones and smartwatches, a reality facilitated by digitization (e.g., European Commission, 2018, OECD, 2018, Ofcom, 2020).

[2] It is well-known that platforms such as Uber and Lyft offer personalized pricing. In addition, other examples of platforms offering personalized pricing to consumers include Target's e-commerce platform, credit card firms such as Mastercard, and travel package firms such as On The Beach, among others.

[3] See FTC order seeking information on surveillance pricing link.

[4] This can be directly observed through FTC observation that firms are increasingly adopting personalized pricing strategies.

addition, recent research suggests that personalized pricing may not consistently yield negative consequences for platforms (e.g., Shaffer and Zhang, 2002, Choudhary et al., 2005, Matsumura and Matsushima, 2015, Esteves and Shuai, 2022). In our paper, we present a mechanism that helps reconcile these contrasting viewpoints and observations on the impact of personalized pricing on platform profitability, competition, and consumers in the presence of network benefits.

Towards this, we consider the effect of personalized pricing in a two-sided market in which content developers and consumers interact on platforms. An apt illustration of such markets includes online platforms such as Uber (both Uber Taxi and UberEats) that offer personalized discounts on purchases linked to your account.[5] Also, the Dutch Competition Ombudsman (Authority for Consumers and Markets (ACM)) found that Wish, an online e-commerce platform, engaged in personalized pricing.[6][7] Given the prevalence of such pricing schemes in platform markets, engaging in a discussion about personalized pricing in a two-sided market is an academic as well as a policy-relevant endeavor.

Let's discuss the two-sided market scenario outlined here. Two competing platforms provide services to both consumers and developers. To illustrate, consider UberEats (Uber Taxi), which facilitates interaction between consumers and restaurants (taxi drivers). Consumers are on a line segment, a Hotelling line, and developers decide whether to affiliate

[5] The following article provides the details of the Uber case in an earlier stage: Uber Testing New Policy: Charge What It Thinks You're Able to Pay (May 22, 2017) (see link). Also, see the Stanford University study on ride-hailing services and price discrimination (see link).

[6] Furthermore, the Austrian Arbeiterkammer (AK - Chamber of Labour) concluded in 2019 that different flight and hotel booking websites showed varying prices depending on whether a computer or a mobile device was used to access the website (see link). More detailed discussions on the prevalence of personalized pricing can be found in a study commissioned by the European Parliament (Rott et al., 2022). See also Bourreau and De Streel (2018) for instances of personalized pricing.

[7] Another example is e-commerce websites such as Target which target consumers based on their locations (see link). Other examples, include On The Beach (a travel booking website), Lyft, Mastercard among others. In addition, one could also consider online subscription-based video-on-demand services. In this context, leading companies like Netflix provide users with personalized recommendations (Kim et al., 2017). The capability to deliver such personalized recommendations suggests that Netflix potentially incorporates personalized pricing into its strategy, as discussed by Shiller (2020). We can extend this thinking to Amazon Prime Video, which similarly excels in providing personalized recommendations.

with a platform according to their outside options. Developers with lower outside options tend to enter the market, and those with higher outside options choose such outside options. The gains of a consumer from participating in one of the platforms are the intrinsic value of the service and the network interaction value that depends on the number of developers participating in this platform. The gain of a developer participating in this platform is the network interaction value that depends on the number of participating consumers. We consider two scenarios: the two platforms use uniform pricing on both sides; they use uniform pricing on the developer side and consumer-side personalized pricing based on preferences for the two platforms. This preference information is available *within* the market, although the platforms need *outside* information about the outside options of developers to offer personalized fees. Therefore, the platforms can employ personalized pricing only for consumers.

We show that personalized pricing charged to consumers benefits developers and can benefit competing platforms compared with uniform pricing. When those multi-sided platforms adopt uniform pricing, consumer prices decrease as the degree of developers' network value increases.[8] Those lower prices enlarge the developers' network value more, allowing platforms to increase their fees on the developer side. The logic works the other way around. Due to the feedback loops of network benefits on both sides, the prices and fees become sufficiently lower as the degrees of consumers' and developers' network values increase. With prices being strategic complements, the rival platforms also lower prices. This increases competition when the network value increases. This feedback loop is partly shut down as personalized pricing, being inherently private, cannot influence developers' expectations regarding the network sizes.[9] The lack of influence prompts the platforms to set higher personalized prices, exploiting consumers' benefits. The degree of rent extraction from consumers increases as the network benefits for consumers increase, resulting in reduced participation fees for developers. However, unlike developers, this extraction

[8] See Rochet and Tirole (2003), Parker and Van Alstyne (2005), Armstrong (2006), Rochet and Tirole (2006), Jullien (2011).

[9] This is also stated by Lindsay Owens, the executive director of Groundwork Collaborative, an economic policy think tank. See the following article for more details (see link).

negatively impacts consumers while benefiting platforms.

This novel and counterintuitive result on the impact of personalized pricing in our paper is elicited solely due to cross-sided network effects. Note that the consumers' network interaction value is not so influential because personalized pricing only slightly influences developer fees. Finally, the total surplus is unambiguously higher under personalized pricing because the number of developers under personalized pricing is larger than under uniform pricing, thanks to lower fees. These findings offer policy insights on personalized pricing.

We perform some robustness checks in the form of extensions to discuss the limitations of our results. We show that our result remains valid under various conditions, including asymmetric platforms, single-homing developers, platforms that invest in developer benefits, some multi-homing developers, and platforms that charge a commission fee.[10]

The rest of the paper is as follows. In Section 2, we discuss the related literature. Following that, in Section 3, we lay down the model. The analysis is presented in Section 4, where we first discuss the outcome under uniform pricing, then explore the outcome under price discrimination, and finally delve into the welfare effects of personalized pricing. Section 5 discusses the robustness of our result to multiple extensions. Section 6 compares our results with Liu and Serfes (2013), a closely related work. We present the policy implications in Section 7. We conclude in Section 8. Proofs are relegated to the Appendix.

2 Related Literature

Our study contributes to the extensive literature on two-sided markets (e.g., Caillaud and Jullien, 2003, Rochet and Tirole, 2003, Parker and Van Alstyne, 2005, Armstrong, 2006, Rochet and Tirole, 2006, Jullien, 2011). In contrast to previous research where uniform prices were charged to consumers, we enhance this stream of literature by incorporating the ability of platforms to set personalized consumer prices – a relevant feature of today's digital market – and analyzing the resulting outcomes.

[10] A detailed analysis of these extensions is available in the Online Appendix.

We also contribute to the extant literature on competitive personalized pricing. Since the seminal works by Thisse and Vives (1988) and Shaffer and Zhang (2002), many studies have investigated the effects of personalized pricing on profits and welfare. The key insight of those studies is that personalized pricing intensifies competition and improves consumer welfare (e.g., Thisse and Vives, 1988, Choe et al., 2018, Houba et al., 2023). In contrast, several studies employ different demand systems and show that personalized pricing benefits firms and harms consumer welfare in static one-sided markets (Liu and Shuai, 2013, Esteves and Resende, 2019, Chen et al., 2020, Esteves, 2022, Esteves and Shuai, 2022, Matsushima et al., 2023, Rhodes and Zhou, 2024, Lu and Matsushima, 2024).[11] Rhodes and Zhou (2024) qualifies the conditions under which the consumer surplus-increasing results in Thisse and Vives (1988) hold. They find that the welfare results in Thisse and Vives (1988) hold only when market coverage is high. Related to Rhodes and Zhou (2024), Lu and Matsushima (2024) also show that personalized pricing harms consumer surplus in a Hotelling duopoly model with multi-item purchasing if the additional utility from the second item is high and consumers are more likely to purchase from both platforms.

In our work, we elucidate the conditions under which personalized pricing can lower consumer surplus even under full market coverage, as in Thisse and Vives (1988).[12] The novelty of our results is a direct consequence of the presence of network effects, which are absent in the related studies mentioned above. Specifically, the private nature of personalized pricing schemes implies they do not contribute to expectation formation. This lowers competition intensity vis-à-vis uniform (public) consumer prices.

Focusing on price discrimination in markets featuring network effects, two closely related papers investigate the effect of personalized pricing in two-sided markets: Liu and Serfes (2013) and Kodera (2015). Liu and Serfes (2013) compare the profits of two plat-

[11] Liu and Shuai (2013) employ a two-dimensional square with inelastic demand, Esteves and Resende (2019) discuss homogeneous goods with informative advertising as in Stahl II (1994), Chen et al. (2020) introduces active consumers, Esteves and Shuai (2022) discuss elastic demands, and Esteves (2022) and Matsushima et al. (2023) introduce heterogeneous consumer types into spatial competition.

[12] Lu and Matsushima (2025) extend Lu and Matsushima (2024) by following the demand system used in Rhodes and Zhou (2024). They show that personalized pricing always improves profits and can either benefit or harm consumer welfare.

forms under uniform and personalized pricing in the duopolistic two-sided Hotelling model described by Armstrong (2006). They consider the case where the platforms employ personalized pricing on both sides, differing from ours (which focuses only on the consumer side). They show that personalized pricing is better for the platforms if and only if the sum of the degree of cross-market externality on consumers and that on participating platforms is higher than a threshold value.[13]

The findings of Liu and Serfes (2013, Section 3.3) rely on the assumption that personalized prices are observable and can be used to credibly commit to offering below-cost prices. They also explore the case where personalized prices are private (Liu and Serfes, 2013, Section 3.5), showing that the price schedules in both cases resemble those in Thisse and Vives (1988). This is because they consider personalized pricing on both sides in the Hotelling model. This implies that the network effect feedback loop is shut down on both sides, and the platforms compete fiercely for consumers on both sides as traditional firms. This leads to an outcome where their results resemble Thisse and Vives (1988) and total surplus remains unchanged.

In contrast to their paper, we consider a competitive bottleneck model where consumer-side single-homing and developers' demands are elastic. We find that personalized pricing can still be profitable even when these prices are secret.[14] Also, we assume that personalized prices are private.[15] We confirm that personalized pricing can still be profitable even when these prices are secret. This difference in market structure elicits interesting welfare results, demonstrating that personalized pricing leads to increased developer participation (and

[13] Kodera (2015) extends Liu and Serfes (2013) by replacing one of the sides in Liu and Serfes (2013) with advertisers that cause negative externality on consumers. He also assumes that the platforms exert personalized pricing only on the advertiser side. He shows that personalized pricing is better for the platforms only if the degree of negative externality on consumers is sufficiently large.

[14] Uniform pricing on the developer side is not essential whenever personalized prices are private.

[15] The private nature of personalized pricing towards consumers is also one of the concerns of policy experts. For instance, Lindsay Owens, executive director of Groundwork Collaborative, an economic policy think tank, states "*surveillance prices erode the longstanding practice of having a public price, which emerged when retailers stopped haggling over everything and started putting price tags on their goods. Public prices are important because they help ensure fairness and are transparent and predictable.*" For more details, see link.

their welfare) as well as higher total welfare. These results differ from those in Liu and Serfes (2013). Furthermore, we provide several extensions to clarify the robustness of our main results.

Our work relates to Hajihashemi et al. (2022), who study a monopoly model with direct network externalities between two consumers with differing reservation values. A key assumption is that personalized prices are private due to their nature, which resembles the passive expectations effect (coordination failure or *Decision Alignment Failure* in Hajihashemi et al. (2022)) described in Hagiu and Hałaburda (2014). Here, consumer expectations are unaffected by prices. This reduces the efficiency of price as a tool to extract surplus. Because of the private nature of personalized pricing, coordination failure is more likely, which lowers the profit of a monopolist. When consumer heterogeneity increases (or the mass of low-value consumers shrinks), the value from network effects weakens, making decision alignment less important. In such cases, price discrimination enhances profit, whereas coordination failure dominates when network effects are strong (see Hajihashemi et al., 2022, Figure 3).

In our setting, *competition* introduces a reversal in results compared to Hajihashemi et al. (2022), driven by the differing effects of observable uniform and private personalized pricing. An observable (consumer) price is significantly lower when the network value for developers increases. This is due to the competitive externality posed by each platform. In contrast, private personalized pricing does not influence developers' expectations regarding the size of the consumer base. This secrecy mitigates competition for consumers, as developers are primarily concerned with the fees they are charged, not the prices consumers pay. This allows the platform to charge higher personalized fees to consumers vis-á-vis uniform pricing, as one tool to affect (developer) expectations is absent. This reduces price competition, and thus personalized pricing benefits platforms but harms consumers when the interaction value of developers is high. This is in contrast to the result in Hajihashemi et al. (2022): personalized pricing is unprofitable if network values are high.

Finally, as personalized prices are private and not observed by developers, our work also advances the strand of literature on how information announcements (including pricing information) in platform markets aid in forming expectations regarding network benefits. Hagiu and Hałaburda (2014) find that competing platforms may prefer not to reveal information, including pricing details, to lower competition.[16] Belleflamme and Peitz (2019b) generalize the model of Hagiu and Hałaburda (2014) and additionally find that results depend on the single- or multi-homing decisions of the two sides and competitive intensity. Similarly, Chellappa and Mukherjee (2021) find that pre-announcement to inform market expectations can be profitable for platforms and depend on the competitive intensity. In contrast to these works, information revelation levels in our setting are affected due to the private nature of personalized pricing. Our work bridges the results in these two different streams of literature and elicits novel results that are counterintuitive to the established findings in each piece of literature.

3 Model

We consider a market with two competing multi-sided platforms denoted by $i = 1, 2$ that connect consumers and developers. On the consumer side, platforms 1 and 2 are at the edges of a Hotelling line, with $x_1 = 0$ and $x_2 = 1$, respectively. The uniform pricing benchmark model is identical to that in (Hagiu and Hałaburda, 2014, Section 4.1), and differs from it in the personalized pricing regime.[17]

Consumers are distributed according to their relative preference x for platform 2 over platform 1. This preference x follows a uniform distribution with unit support, i.e., $x \sim \mathcal{U}[0, 1]$. A consumer of type x incurs a mismatch cost of tx and $t(1-x)$ when transacting, respectively, at platforms 1 and 2, where t is a positive constant representing the degree of preference mismatch.[18] The utility of a consumer of type x when purchasing platform 1's

[16] A related work where users cannot observe fees is Ding and Wright (2017).

[17] Reisinger (2012) considers a media competition in a similar framework.

[18] The parameter t is often used as a proxy for a lack of competition. Specifically, as t increases, competition between the two platforms becomes less intense, as consumers closer to one platform find the product of the other platform relatively less valuable to consider.

product or platform 2's product is given as:

$$
\begin{aligned}
U_1(p_1, D_1^e, x) = &\quad w + \theta D_1^e - p_1 - tx, && \text{purchasing from platform 1,} \quad (1)\\
U_2(p_2, D_2^e, x) = &\quad w + \theta D_2^e - p_2 - t(1-x), && \text{purchasing from platform 2,} \quad (2)
\end{aligned}
$$

where $w(> 0)$ is the common intrinsic utility that a consumer enjoys from the consumption of the product. Additionally, θD_i^e represents the expected value consumers derive from interacting with developers, where D_i^e is the expected mass of developers at platform i, and $\theta > 0$ is the interaction value consumers place on interaction with each additional developer at platform i. Furthermore, p_i is the consumer price charged by platform i.[19] Here, the superscript e indicates consumers' expectations for the mass of developers at platform i. Thus, θD_i^e reflects the cross-market network benefit enjoyed by consumers.

Developers in our setting are distributed according to their outside option k, which follows a uniform distribution with unit support, i.e., $k \sim \mathcal{U}[0, 1]$. These developers value interactions with consumers in a platform. The payoff of each developer interacting with consumers at platform $i \in \{1, 2\}$ is

$$\pi_i^{Dev}(l_i, N_i^e) = \phi N_i^e - l_i,$$

where ϕ is the interaction value developers place on interacting with an additional consumer, N_i^e is the expected mass of consumers at platform i, and l_i is the participation fee charged by platform i to developers for interacting with its consumers. The developer of type k participates if and only if[20]

$$\pi_i^{Dev}(l_i, N_i^e) - k \equiv \phi N_i^e - l_i - k \geq 0.$$

Our results are robust to the case where the platform charges commission fees to developers

[19] The price can be seen as the subscription fee that Uber charges for its premium service Uber One.

[20] Note that here we differ from Liu and Serfes (2013) as the developer demand is elastic in our setting. Instead, in Liu and Serfes (2013), demands on both sides are inelastic. This difference offers more nuanced results on total surplus while total surplus in Liu and Serfes (2013) remains unchanged in the two pricing regimes.

instead of participation fees.[21] In favor of brevity and being close to the canonical models of two-sided platforms, we assume that the platforms charge participation fees to developers.

We assume that detecting each developer's outside value is challenging for the platforms because it requires information about developers' outside opportunities across multiple markets in which they are active. Identifying these opportunities is harder than assessing consumer preferences in a particular market, where platforms can focus on market-specific data. Thus, it is significantly harder for platforms to offer personalized fees to developers. In addition, developers possess highly privileged data, and with the current antitrust scrutiny on platforms, they are careful to avoid being in a situation where they are seen as collecting personal data on sellers they host.[22]

The profit of each platform i is a composite term of consumer sales revenues and developer sales revenues and is given as

$$\Pi_i = \underbrace{p_i N_i}_{\substack{\text{Consumer} \\ \text{sales revenue}}} + \underbrace{l_i D_i.}_{\substack{\text{Developer} \\ \text{sales revenue}}}$$

We consider two consumer pricing regimes employed by platforms: (i) uniform pricing and (ii) personalized pricing. In case (ii), platforms can perfectly identify the locations of all consumers and offer personalized prices to them. That is, prices become a function of consumer types x and are denoted as $p_i(x)$.

[21] In this alternative setting with commission fees, we interpret ϕ as the per-consumer revenue that a developer earns from a transaction with a consumer. Per transaction revenue for a platform is $r_i\phi$, where r_i is the commission fee rate. In essence, the per consumer revenue of each developer is exogenous (does not depend on the commission fees) (see Anderson and Bedre-Defolie (2025) on the microfoundation for this case). This can be seen in markets where developers incur zero or low marginal costs of serving consumers, such as on software platforms, music streaming platforms. In general, commission fees could affect the revenue per consumers if the developer passes these fees on their price to consumers. Nevertheless, we expect that our result holds when the pass through of commission fee into prices is sufficiently low. A detailed analysis of this case is available in the Online Appendix.

[22] We obtain qualitatively similar results in a setting where platforms charge private personalized fees to developers based on information about their outside option k, while simultaneously applying private personalized pricing to consumers. In this setting, however, each developer's net surplus becomes zero in equilibrium, as each platform can charge $\phi N_i^e - k$ to fully extract the surplus from developers with outside option k. In our benchmark, we focus on a less extreme case where platforms charge uniform fees to developers.

The timing of the game is as follows:

(1.) Platforms simultaneously offer prices p_i and fees l_i to consumers and developers, respectively. When platforms employ personalized pricing, which is private, we replace p_i with $p_i(x)$.

(2.) Consumers and developers, respectively, form expectations for the masses of developers and consumers in each platform and then decide to affiliate with platforms. Subsequently, profits are realized.

We impose the following technical assumptions.

Assumption 1 *(i) The intrinsic value is high enough — i.e., $w \geq 3t/2$. (ii) The exogenous parameters, t, θ, and ϕ, satisfy $t > \underline{t} \equiv \max\{(\theta^2 + 6\theta\phi + \phi^2)/8, \theta(\theta + 6\phi + \sqrt{\theta^2 + 12\theta\phi + 4\phi^2})/8\}$.*

The first restriction ensures that each consumer purchases at one of the platforms. The second assumption ensures that the second-order conditions are satisfied in both pricing regimes.

4 Analysis

We consider two cases in which platforms employ the following pricing schemes on the consumer side: (i) uniform pricing and (ii) personalized pricing. We then compare the outcomes in the two cases and present the welfare results.

4.1 Uniform pricing

In stage 2, we first derive the demand functions to formulate the objectives of the platforms. Observing the prices and fees set in stage 1, consumers and developers form expectations D_i^e and N_i^e and then participate in platform i.

From equations (1) and (2), we derive the location of indifferent consumers denoted by $\bar{x}$, which provides us with the mass of consumers at platform 1 and platform 2 as:

$$N_1(D_1^e, D_2^e, p_1, p_2) = \bar{x} = \frac{t + \theta(D_1^e - D_2^e) - p_1 + p_2}{2t}, \ N_2(\cdot) = 1 - \bar{x}. \tag{3}$$

The above demands are intuitive: as the consumer price set by platform i rises, their demand for the product of platform i falls. Conversely, as the price set by the rival platform $-i$ increases, the demand at platform i rises. Additionally, as the expectation of the value from developer interaction (θD_i^e) increases, consumer demand at platform i rises as well.

Developers participate on the platform as long as they enjoy positive payoffs, i.e., $\pi_i^{Dev}(\cdot) \geq k$. Solving this inequality yields the indifferent developer type's outside option denoted by $\bar{k}_i$, below which developers find it profitable to participate on platform i. Thus, we can express the mass of developers active on platform i as:

$$D_i(N_i^e, l_i) = \bar{k}_i(N_i^e, l_i) = \phi N_i^e - l_i. \tag{4}$$

Because we employ a fulfilled expectations equilibrium, the expected mass of developers and consumers must match the realized demands. By imposing $N_i^e = N_i^\star$ and $D_i^e = D_i^\star$ in equations (3) and (4) and solving for the mass of consumers and the mass of developers on two platforms, we obtain demands as a function of prices and fees, as presented below for $(i = 1, 2)$:

$$\begin{aligned} N_i^\star(p_i, p_{-i}, l_i, l_{-i}) &= \frac{1}{2} - \frac{(p_i - p_{-i}) + \theta(l_i - l_{-i})}{2(t - \theta\phi)}, \\ D_i^\star(l_i, l_{-i}, p_i, p_{-i}) &= \frac{\phi}{2} - \frac{\phi(p_i - p_{-i}) + (2t - \theta\phi)l_i - \theta\phi l_{-i}}{2(t - \theta\phi)}. \end{aligned} \tag{5}$$

As the consumer price or developer fee at platform i increases, both consumer and developer participation falls. This is because, apart from lowering the direct value of participation on platform i, a higher price or fee also lowers the expected value of interactions on the other side. Furthermore, the equations in (5) show that as the degree of network benefits rises, demands become more price-elastic.

In stage 1, each platform $i \in \{1, 2\}$ sets prices and fees to maximize its profits, given as

$$\max_{l_i, p_i} \Pi_i(p_i, p_{-i}, l_i, l_{-i}) = p_i N_i^\star(\cdot) + l_i D_i^\star(\cdot).$$

Differentiating the profit of each platform $i \in \{1, 2\}$ with respect to p_i yields the following first-order condition.

$$\underbrace{N_i^\star(\cdot) + p_i \frac{\partial N_i(\cdot)}{\partial p_i}}_{\substack{\text{Volume+} \\ \text{Margin effect}}} + \underbrace{p_i \left[\frac{\partial N_i(\cdot)}{\partial D_i^e} \frac{\partial D_i^\star(\cdot)}{\partial p_i} + \frac{\partial N_i(\cdot)}{\partial D_{-i}^e} \frac{\partial D_{-i}^\star(\cdot)}{\partial p_i} \right]}_{\text{Consumer participation effect } (-)} + \underbrace{l_i \frac{\partial D_i(\cdot)}{\partial N_i^e} \frac{\partial N_i^\star(\cdot)}{\partial p_i}}_{\substack{\text{Developer participation} \\ \text{effect } (-)}} = 0. \tag{6}$$

The above first-order condition describes the marginal impact of an increase in consumer price on the profitability of platform i. The first two terms represent the classical volume and margin effects.

The second effect demonstrates how a unit (consumer) price increase affects consumers' participation through changes in their expectations regarding developer participation. Specifically, an increase in price p_i lowers consumers' expectations regarding the participation of developers on platform i. Similarly, consumers' expectations regarding developer participation at the rival platform $-i$ increase. These two effects reinforce each other and lower consumer demand at platform i with a unit increase in price p_i, arising because platforms directly compete for consumers.

The final expression represents how a unit increase in consumer price affects developer participation on the platform through changes in their expectations of the value derived from consumer participation. Specifically, an increase in price p_i lowers developers' expectations regarding the participation of consumers on platform i, subsequently reducing their own participation as well. A direct consequence of these reinforcing effects, which lower platform profitability, is that platforms will compete more fiercely when setting consumer prices, compared to the case without network effects.

Differentiating the profit of each platform $i \in 1, 2$ with respect to l_i yields the following first-order condition.

$$\underbrace{D_i^\star(\cdot) + l_i \frac{\partial D_i(\cdot)}{\partial l_i}}_{\substack{\text{Margin+} \\ \text{Volume effect}}} + \underbrace{p_i \left[\frac{\partial N_i(\cdot)}{\partial D_i^e} \frac{\partial D_i^\star(\cdot)}{\partial l_i} + \frac{\partial N_i(\cdot)}{\partial D_{-i}^e} \frac{\partial D_{-i}^\star(\cdot)}{\partial l_i} \right]}_{\text{Consumer participation effect } (-)} + \underbrace{l_i \frac{\partial D_i(\cdot)}{\partial N_i^e} \frac{\partial N_i^\star(\cdot)}{\partial l_i}}_{\substack{\text{Developer participation} \\ \text{effect } (-)}} = 0. \tag{7}$$

As in the above, a similar discussion regarding the impact of a unit increase in developer participation fees on the profitability of the platforms can be easily made.[23]

[23] Similar models appear in works such as Hagiu and Hałaburda (2014) and Shekhar (2021). For clarity,

Solving the system of first-order conditions in equations (6) and (7) yields the equilibrium fees, as presented below.

$$p_i^U = t - \frac{\phi(3\theta + \phi)}{4}, \quad l_i^U = \frac{\phi - \theta}{4}. \tag{8}$$

First, note that, in comparison to a traditional Hotelling model without network effects, the uniform consumer price is lower. This is due to network effects, which encourage platforms to set low consumer prices unilaterally. Furthermore, recalling that prices are strategic complements, the rival platform also lowers prices, resulting in fiercer competition with increased value from network effects. On the developer side, given that each platform has monopoly power over developers, an increase in the degree of network interaction benefits, ϕ, raises the fee for developers. At the same time, the consumer price, p_i, decreases to enhance the network benefits on developers, ϕN_i. Contrary to the effect of ϕ on fees and prices, as the degree of network benefits on consumers, θ, becomes larger, the fee for developers, l_i , and the consumer price, p_i, decrease due to elastic developer demand and the incentives to enhance the network benefits on consumers, θD_i.

Substituting the equilibrium prices and fees presented in equation (8) into the calculations for profits, demands, consumer surplus, and producer surplus yields the following outcome:

Lemma 1 *The equilibrium profits, the equilibrium mass of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:*

$$\begin{aligned} \pi_i^U &= \frac{t}{2} - \frac{\theta^2 + 6\theta\phi + \phi^2}{16}, \quad N_i^U = \frac{1}{2}, \quad D_i^U = \frac{\theta + \phi}{4}, \\ CS^U &= w - \frac{5t}{4} + \frac{\theta^2 + 4\theta\phi + \phi^2}{4}, \quad PS^U = \frac{(\theta + \phi)^2}{16}. \end{aligned}$$

Platform profits monotonically decrease with the increase in the values of network interactions θ and ϕ. This is because each platform now independently finds it profitable to expand the network. As platforms compete, their strategic interaction leads to accelerated competition. As competition intensifies, both consumer surplus and developer surplus increase with the degree of network benefits, θ and ϕ.

we present this case in greater detail to facilitate comparisons of the pricing regimes.

4.2 Personalized pricing

In personalized pricing, the two competing platforms can perfectly identify consumers based on their type x and establish individualized pricing schedules.[24]

We assume that personalized prices are kept confidential, known only to the involved parties, and do not influence the expectations developers form regarding consumer participation on platform i (N_i^e for $i = 1, 2$). In reality, disclosing individual trading terms to the public could raise privacy concerns; hence, we make this assumption, as in Hajihashemi et al. (2022).

We derive the results when platforms can identify consumers and implement personalized pricing. Unlike in the case of uniform pricing, platforms do not offer negative (personalized) prices to consumers because such consumer prices are private and, therefore, do not affect the expected mass of consumers N_i^e on each platform. Negative personalized prices just lead to a loss in platform profits, and therefore, the lowest personalized price becomes zero. Considering the nature of personalized pricing and the utilities presented in (1) and (2), we obtain the price schedules of platforms 1 and 2 when the rival platform sets a zero price. This involves determining the location of indifferent consumers and, consequently, the mass of consumers on platform i:

$$p_1(x) = \begin{cases} \theta(D_1^e - D_2^e) + t(1-2x) & \text{if } x \le \bar{x}, \\ 0 & \text{if } x > \bar{x}, \end{cases} \tag{9}$$

$$p_2(x) = \begin{cases} 0 & \text{if } x \le \bar{x}, \\ \theta(D_2^e - D_1^e) + t(2x-1), & \text{if } x > \bar{x}. \end{cases} \tag{10}$$

$$N_1(D_1^e, D_2^e) = \bar{x}(D_1^e, D_2^e) = \frac{t + \theta(D_1^e - D_2^e)}{2t}, \quad N_2(\cdot) = 1 - \bar{x}(\cdot).$$

Considering the fees charged to developers, the expected number of developers must align with the actual number, denoted as $D_i^e = D_i^{\star\star}$ $(i = 1, 2)$, under the conditions $N_i^e = N_i^{\star\star}$. By employing the expression for consumer demand above and the expression for developer demand as in equation (4), and then solving, we find that the mass of developers

[24] This extreme information structure is commonly employed in related works on personalized pricing (e.g., Thisse and Vives, 1988, Shaffer and Zhang, 2002, Esteves and Shuai, 2022)

active in each platform i is solely influenced by the developer participation fees:

$$D_i^{\star\star}(l_i, l_{-i}) = \frac{\phi}{2} - \frac{(2t - \theta\phi)l_i - \theta\phi l_{-i}}{2(t - \theta\phi)} \text{ for } i = 1, 2. \tag{11}$$

As with uniform pricing, when the developer fee on platform i increases, developer participation on platform i decreases. Additionally, an increase in the fee on platform i also decreases consumer demand on platform i by decreasing the network benefit θD_i.

As consumer prices are not observed and remain private, the mass of developers, denoted as $D_i^{\star\star}(\cdot)$, is independent of these personalized consumer price schedules. Specifically, as prices are confidential and personalized, platforms lack the ability to utilize these price schedules to influence the participation of developers. This diminishes competition for consumers, as platforms are unable to deploy one of their strategic tools in stage 1 to impact developer participation. The competition mitigation effect resulting from secret (unobserved) consumer prices aligns with the findings in Hagiu and Hałaburda (2014). However, in contrast to Hagiu and Hałaburda (2014), this competition mitigation effect is counteracted by the *competition-enhancing effect* of personalized prices, as discussed in Thisse and Vives (1988). This distinctive aspect of our work introduces nuanced insights into welfare considerations.

By substituting the mass of developers as presented in equation (11) into equation (9), we can derive the actual price schedules and the mass of consumers for platform i as a function of developer fees:

$$p_1^{\star\star}(l_1, l_2, x) = \begin{cases} \dfrac{\theta(l_2 - l_1)t}{t - \theta\phi} + t(1 - 2x) & \text{if } x \leq \bar{x}^{\star\star}, \\ 0 & \text{if } x > \bar{x}^{\star\star}, \end{cases} \tag{12}$$

$$p_2^{\star\star}(l_2, l_1, x) = \begin{cases} 0 & \text{if } x \leq \bar{x}^{\star\star}, \\ \dfrac{\theta(l_2 - l_1)t}{t - \theta\phi} + t(2x - 1), & \text{if } x > \bar{x}^{\star\star}, \end{cases} \tag{13}$$

$$N_1^{\star\star}(l_1, l_2) = \frac{t - \theta\phi + \theta(l_2 - l_1)}{2(t - \theta\phi)}, \quad N_2^{\star\star}(l_2, l_1) = \frac{t - \theta\phi + \theta(l_1 - l_2)}{2(t - \theta\phi)}, \tag{14}$$

where $\bar{x}^{\star\star}(l_1, l_2) = \bar{x}_1(D_1^{\star\star}(\cdot), D_2^{\star\star})$. The price schedules in (12) and (13) suggest that each platform can charge higher personalized prices by reducing its fee to increase the number of developers. The effectiveness of the fee reduction is more pronounced with higher degrees of network benefits, θ.

In stage 1, each platform i strategically determines the value of l_i to maximize its profits.

$$\max_{l_1} \Pi_1^{\star\star} = \int_0^{\bar{x}^{\star\star}} p_1^{\star\star}(l_1, l_2, x)dx + l_1 D_1^{\star\star}(\cdot), \quad \max_{l_2} \Pi_2^{\star\star} = \int_{\bar{x}^{\star\star}}^1 p_2^{\star\star}(l_2, l_1, x)dx + l_2 D_2^{\star\star}(\cdot).$$

Employing the Leibniz integral rule and differentiating the profit of platform 1 with respect to the fee l_1 yields

$$\begin{aligned} &\underbrace{D_1^{\star\star}(\cdot) + l_1 \frac{\partial D_1(\cdot)}{\partial l_1}}_{\substack{\text{Margin + Volume} \\ \text{effect}}} + \underbrace{l_1 \frac{\partial D_1(\cdot)}{\partial N_1^e} \frac{\partial N_1^\star(\cdot)}{\partial l_1}}_{\substack{\text{Developer participation} \\ \text{effect } (-)}} \\ &+ \underbrace{\int_0^{\bar{x}^{\star\star}} \left[\underbrace{\frac{\partial p_1(x)}{\partial D_1^e}}_{(+)} \underbrace{\frac{\partial D_1^{\star\star}(\cdot)}{\partial l_1}}_{(-)} + \underbrace{\frac{\partial p_1(x)}{\partial D_2^e}}_{(-)} \underbrace{\frac{\partial D_2^{\star\star}(\cdot)}{\partial l_1}}_{(+)} \right] dx}_{\text{Personalized price effect } (-)} = 0. \end{aligned} \tag{15}$$

The terms in the first line of the above first-order expression mirror those in the uniform pricing case (as in equation (7)). The term in the second line introduces a novel effect, elucidating how observable developer participation fees influence the (private) personalized prices charged to consumers. To elaborate, consider that the personalized price for each consumer type x is established to extract the entire *consumer arbitrage value* from purchasing at platform i, given that the rival platform $-i$ sets a zero price. This arbitrage value is contingent upon the difference in expected interaction value. A unit increase in developer participation fee l_i detrimentally impacts this expected interaction value, consequently influencing the private personalized price. This new effect adversely affects the first-order condition, prompting each platform to unilaterally set lower developer participation fees. It's crucial to note that the incentive to set lower developer fees due to network effects emerges here as well, albeit through a distinct mechanism than in the uniform pricing case.[25]

[25] Note that this effect is absent in Liu and Serfes (2013) even when they consider private personalized

Similarly, we can obtain the first-order condition with respect to l_2 for platform 2. Solving the system of fist-order conditions presented in equation (15) yields the equilibrium fees as follows.

$$l_1^P = l_2^P = \frac{t(\phi - \theta) - \theta\phi^2}{4t - 3\theta\phi}. \tag{16}$$

Substituting these equilibrium fees into the personalized pricing schedules yields

$$p_1^P(x) = \begin{cases} t(1-2x) & \text{if } x \leq 1/2, \\ 0 & \text{if } x > 1/2, \end{cases} \quad p_2^P(x) = \begin{cases} 0 & \text{if } x \leq 1/2, \\ t(2x-1), & \text{if } x > 1/2. \end{cases} \tag{17}$$

The price schedules, $p_i^P(x)$, in (17) are identical to those in Thisse and Vives (1988).

Substituting the equilibrium fees as in equations (16) and (17) yields the following outcome:

Lemma 2 *The equilibrium profits, the equilibrium numbers of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:*

$$\begin{aligned} \Pi_i^P &= \frac{t}{4} - \frac{((\theta - \phi)t + \theta\phi^2)(2(\theta+\phi)t - \theta\phi^2)}{2(4t - 3\theta\phi)^2}, \quad N_i^P = \frac{1}{2}, \quad D_i^P = \frac{2(\theta+\phi)t - \theta\phi^2}{2(4t-3\theta\phi)}, \\ CS^P &= w - \frac{12t^2 - \theta(4\theta + 13\phi)t + 2\theta^2\phi^2}{4(4t - 3\theta\phi)}, \quad PS^P = \frac{(2(\theta+\phi)t - \theta\phi^2)^2}{4(4t-3\theta\phi)^2}. \end{aligned}$$

Although the profits are decreasing in the value of network interactions, CS^P and PS^P are increasing in θ and ϕ.

4.3 Welfare effects of pricing regimes

We compare platform prices, fees, profits, consumer surplus, developer surplus, and the total surplus across the two regimes.

Before proceeding to compare prices and fees in the two regimes, it is essential to establish a statistic facilitating the comparison. Given that prices in personalized pricing form a menu contingent on consumers' location, as discussed in Thisse and Vives (1988), we opt for the average price faced by consumers in personalized pricing as the key statistic

prices. This is because they consider personalized pricing on both sides, which shuts down the network effects feedback loop.

for comparison. The average price in personalized pricing is defined as:

$$\mathbb{E}p^P = \int_0^{N_1^P} p_1^P(x)dx + \int_{N_1^P}^1 p_2^P(x)dx = \frac{t}{2}.$$

Proposition 1 *The average consumer price in personalized pricing is higher than the price in uniform pricing when $\phi \geq \widehat{\phi} := (\sqrt{8t + 9\theta^2} - 3\theta)/2$. Developer fees are unambiguously lower in personalized pricing than in uniform pricing — i.e., $l_i^P < l_i^U$.*

The first statement of the proposition confirms the findings under observable personalized prices in Liu and Serfes (2013, Section 3.3). The subsequent portion of the proposition then presents a comparative static analysis as outlined in Kodera (2015, Section 3), despite their lack of explicit discussion on the relationship.

Consistent with Thisse and Vives (1988), our findings reveal that when the developer interaction value is low, the average consumer price in personalized pricing, $t/2$, is lower than that in uniform pricing (see p_i^U in (8)). Interestingly, in contrast to Thisse and Vives (1988), we observe that consumer prices in personalized pricing can surpass those in uniform pricing when the degree of network interactions enjoyed by developers, denoted as ϕ, is substantial. This distinctive result emerges solely due to the presence of cross-sided network interactions, which are absent in Thisse and Vives (1988).

These results demonstrate how the competition-enhancing effects of personalized pricing interact with the competition-dampening effects due to consumer prices being private under personalized pricing. When the developers' network interaction value is low, cross-sided network effects do not significantly impact the outcome, leading to the traditional result that personalized pricing enhances competition, thereby lowering consumer prices. Conversely, when the developers' value from network interactions is sufficiently high, the consumer price under uniform pricing decreases. This is attributed to the fact that decreased (public) uniform consumer prices also attract more developers (as they expect a greater mass of consumers on the platform), who are then charged a higher fee. Specifically, under the uniform pricing regime, consumer prices decrease with an increase in ϕ as each platform unilaterally finds it profitable to expand cross-market network values for developers. As platforms engage in competition and prices are strategic complements, this strategic effect contributes to a further reduction in uniform prices. In contrast, the average consumer price (see above) under personalized pricing remains independent of the network interaction values for developers, denoted as ϕ, and therefore remains unchanged with variations in ϕ. Consequently, when ϕ is sufficiently high, the consumer prices in uniform pricing become lower than the average price in personalized pricing.

The fees in personalized pricing denoted as l_i^P, are unequivocally lower than those in uniform pricing, represented as l_i^U. This distinction arises from the fact that platforms are unable to leverage consumer prices to influence the mass of developers; they can only use fees charged to developers for this purpose. Consequently, platforms strategically set low fees to attract developers, a strategy that proves beneficial for establishing high personalized consumer prices (refer to (12) and (13)).

Next, we aim to understand the effects of personalized pricing on the developer surplus, PS, and the total surplus, TS. In pricing regime $k \in \{U, P\}$, we define the total surplus as the sum of platform profits (denoted by Π_i), consumer surplus (denoted by CS), and developer surplus (denoted by PS).

$$TS^k = \sum_{i=1}^{2} \Pi_i^k + CS^k + PS^k \text{ for } k \in \{U,\ P\}.$$

Under Assumptions 1, we obtain the following outcomes, which are summarized as in Proposition 2:

$$\Delta PS = PS^P - PS^U = \frac{\theta\phi(3\theta + \phi)(8(\theta + \phi)t - \theta\phi(3\theta + 5\phi))}{16(4t - 3\theta\phi)^2} > 0, \tag{18}$$

$$\Delta TS = TS^P - TS^U = \frac{\theta\phi(3\theta + \phi)(8(\theta + \phi)t - \theta\phi(9\theta + 7\phi))}{16(4t - 3\theta\phi)^2} > 0. \tag{19}$$

Proposition 2 *Personalized pricing improves the surplus of developers and the total surplus.*

As personalized pricing prompts platforms to lower developer fees l_i, the developers' surplus under personalized pricing surpasses that under uniform pricing. Additionally, these reduced fees for developers contribute to the expansion of network benefits for consumers under personalized pricing. This expansion results in a higher total surplus under personalized pricing compared to uniform pricing. The welfare implications of personalized pricing differ from those in Liu and Serfes (2013) and Kodera (2015) due to the inelastic demands on both sides (in their setting), as seen in the standard Hotelling model.

Here, we examine the effects of personalized pricing on profits and consumer surplus. To do so, we define the difference in profits and consumer surplus between the two regimes as $\Delta\Pi = \Pi_i^P - \Pi_i^U$ and $\Delta CS = CS^P - CS^U$. Figure 1 illustrates the regions in which personalized pricing enhances profits or consumer surplus.

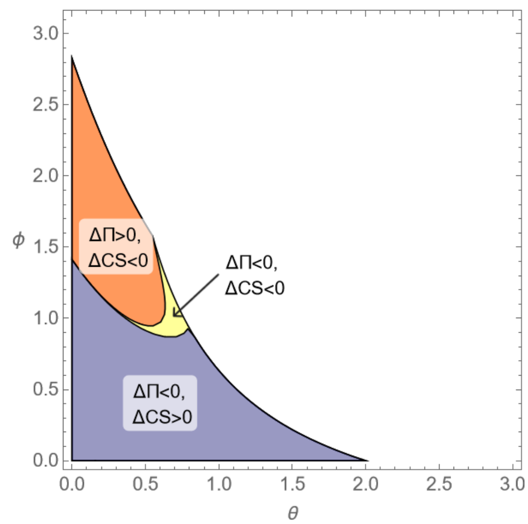


Figure 1: Comparison of profits and consumer surplus ($t = 1$ and $v = 3/2$).

Proposition 3 *The following relationship holds.*

- *When $0 < \phi < \phi_1$, consumer surplus (platform profit) under personalized pricing is higher (lower) higher than under uniform pricing — i.e., $\Delta CS > 0$ and $\Delta\Pi < 0$.*

- *When $\phi_1 < \phi < \phi_2$, consumer surplus and platform profits under personalized pricing are lower than under uniform pricing — i.e., $\Delta CS < 0$ and $\Delta\Pi < 0$.*

- *When $\phi_2 < \phi$, consumer surplus (platform profit) under personalized pricing is lower (higher) than under uniform pricing — i.e., $\Delta CS < 0$ and $\Delta\Pi > 0$.*

When the developer interaction value is sufficiently low, the platform faces challenges in attracting an adequate number of developers, even with lowered fees under personalized prices. Additionally, under this parameter configuration, the dominance of the competition-enhancing effect of personalized pricing results in lower consumer prices compared to uniform prices. A direct consequence of both consumer prices and developer fees being lower is a decrease in platform profits under personalized prices. Consequently, the personalized pricing regime, despite increasing the mass of participating developers, adversely impacts platforms when $\phi < \phi_1$.

When the developer interaction value is intermediate, both platforms and consumers experience adverse effects. Platform profits decline as they reduce fees to developers but struggle to establish sufficiently high consumer prices due to the intensified competition effect, resulting in lower profits. Consumers also face a disadvantage as the benefits from increased interaction with developers under personalized prices are overshadowed by the (relatively) higher average prices charged to them. In this region, only developers benefit from personalized pricing.

When the developer interaction value is high, platforms are better off under personalized pricing. However, consumer welfare is worse under personalized pricing, where the competition-reducing effect dominates the competition-increasing effect. This is evident from the discussion following Proposition 1, where the average consumer price is higher under personalized pricing when ϕ is sufficiently high. Consequently, platform profits increase under personalized pricing, although consumer surplus declines.

Our results demonstrate both similarities and differences when compared to the findings presented by Liu and Serfes (2013). In particular, Liu and Serfes (2013) assert that personalized pricing is advantageous for platforms if and only if the sum of the degree of cross-market externality on consumers and participating platforms exceeds a threshold value. While this conclusion implies that the effect of the degree of cross-market externality on consumers is equivalent to that on participating platforms, our result, though partially resembling theirs, emphasizes that the degree of cross-market externality on developers is more critical than that on consumers in our model. In addition, we find that the total surplus increases under personalized pricing while in their setting total surplus remains unchanged due to inelastic demand on both sides.

5 Extensions

In the following, we present three extensions that discuss the robustness of our result as well as the limitations.

5.1 Asymmetric platforms

We extend the model by considering quality asymmetry between the platforms. We modify the utility when a consumer purchases from platform 1 as follows (see (1)): $U_1(p_1, D_1^e, x) = w + h + \theta D_1^e - p_1 - tx$, where $h(> 0)$ is the quality advantage of platform 1. This asymmetry can be understood as the relative market advantage of a dominant platform vis-á-vis its rival.

We discuss the effect of platform asymmetry on the platform profits, consumer surplus, and total surplus. A simple calculation leads to the following proposition:

Proposition 4 *ΔCS is always decreasing in h. ΔTS is increasing in h if and only if $\phi < \phi_3$, where ϕ_3 is an upper bound of ϕ. Also, $\phi_3 > \phi_2$, where ϕ_2 is in Proposition 3.*

First, ΔCS is decreasing in h because the dominant platform attracts many consumers and exploits their surpluses through personalized pricing. We can check the surplus extraction through personalized pricing by comparing the prices under uniform and personalized pricing. The (average) prices of platform 1 under uniform and personalized pricing, and the difference between them are

$$\begin{aligned}
p_1 &= t - \frac{\phi(3\theta+\phi)}{4} + \frac{4t - \phi(3\theta+\phi)}{2(6t - \theta^2 - 4\theta\phi - \phi^2)}h, \\
E[p_1(x)] &= \underbrace{\frac{t}{2}}_{E[t(1-2x)]} + \frac{t(4t - 3\theta\phi)}{4t^2 - \theta(2\theta + 5\phi)t + \theta^2\phi^2}h, \\
E[p_1(x)] - p_1 &= \frac{\phi(3\theta+\phi)}{4} - \frac{t}{2} + \frac{4t^2(8t - \phi(9\theta+\phi)) + \theta\phi^2(3\theta+\phi)(t+\theta\phi)}{2(6t - \theta^2 - 4\theta\phi - \phi^2)(4t^2 - \theta(2\theta+5\phi)t + \theta^2\phi^2)}h.
\end{aligned}$$

The coefficient of h in $E[p_1(x)] - p_1$ is positive and increasing in θ and ϕ, implying that the per-consumer payment under personalized pricing becomes higher than that under uniform pricing as the advantage of platform 1 strengthens and the cross-market externalities increase. The intuition for this is as follows: Prices are strategic complements under uniform pricing. A decrease in the uniform price of platform 2 limits an increase in the uniform price of platform 1 due to strategic complementarity. Instead under personalized pricing, prices are strategically independent. Therefore, when platform 1 wins, it extracts its advantage h through the personalized prices.[26] As a consequence, the magnitude of price increment through an increase in h is higher under personalized pricing. This positive relationship between $E[p_1(x)] - p_1$ and h captures the intuition explained here. Thus, we conclude that if a dominant platform exists in a two-sided market, we should be cautious about personalized pricing, as it is more likely to harm consumer welfare.

Second, we briefly discuss the effect of platform asymmetry on the total surplus. Personalized pricing achieves efficient allocations on the consumer side because the efficient platform, at each specific point, provides consumers with superior offerings. The allocation efficiency improves as the value of h increases. However, the superiority of platform 1 under personalized pricing enables it to exert monopoly power over developers. This monopoly power grows stronger as the interaction value for developers, ϕ, increases. Therefore, as the value of h increases, the total surplus under personalized pricing is more likely to increase unless ϕ is sufficiently large (see Figure 2).

[26] The mathematical detail is available in equation (20) in the Appendix.

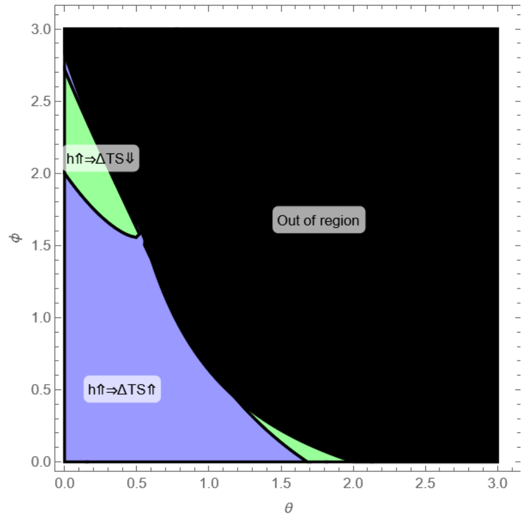
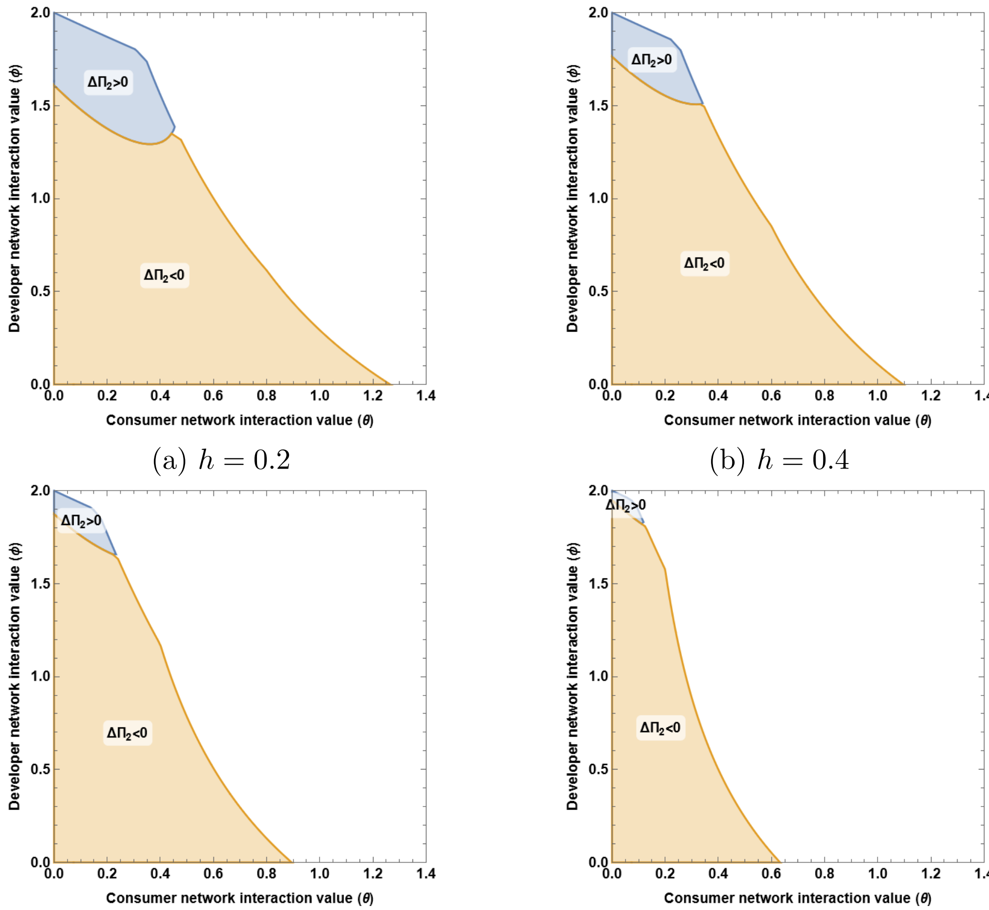


Figure 2: The impact of a marginal increase in h on ΔTS ($t = 1$).

Corollary 1 *As the advantage of the (dominant) platform* 1 *increases (as* h *increases), platform* 1 (2) *is more (less) likely to benefit from personalized pricing.*

The above corollary presents the result that personalized pricing reinforces the advantage of the dominant platform. The rationale for this result is as follows. As the advantage of the dominant platform increases, it is able to attract a larger mass of consumers to its platform through personalized pricing. Anticipating this, more developers also affiliate with the (dominant) platform 1 than the disadvantaged platform 2. As a result, platform 1 is further able to attract consumers closer to platform 2. This feedback loop is reinforced with an increase in the advantage h. This leads to the outcome that the disadvantaged platform is less likely to benefit from personalization as the level of its disadvantage increases (See Figure (3)). This result has important policy implications.



(a) $h = 0.2$ (b) $h = 0.4$

(c) $h = 0.6$ (d) $h = 0.8$

Figure 3: Profitability of platform 2 as h changes for $t = 1$.

5.2 Single-homing developers

In this extension (presented in detail in the Online Appendix), we consider the case where developers single-home at either platform. Towards this, we model a setting where developers have a preference for one platform over the other, as in the Hotelling competition. In such a setting, we find that our main result on platform profits, consumer surplus, and producer surplus hold qualitatively (See Figure 4). The total surplus in the two regimes stays the same (in the two pricing regimes), and this arises from the Hotelling competition on the developers' side, with the demand being covered.

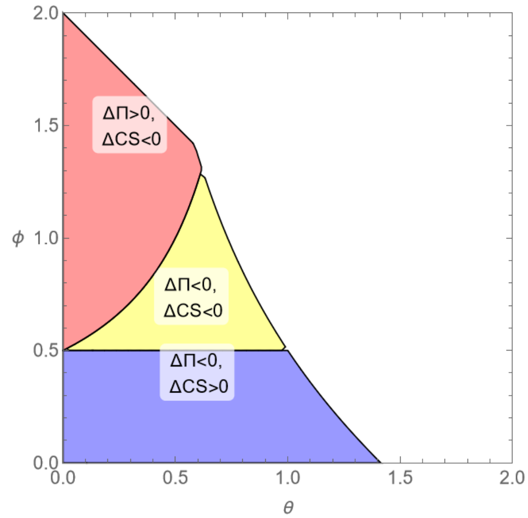


Figure 4: Comparison of profits and consumer surplus ($t = 1$).

5.3 Multi-homing developers

In this extension (presented in detail in the Online Appendix), we consider the case where developers multi-home at both platforms.[27] On the developers' side, incurring fixed costs of k once is enough for developers to participate on both platforms. In this case, we find that our main results do not hold. This is because when all developers multi-home, from the perspective of consumers when deciding which platform to affiliate with, the expected mass of developers on the platform is irrelevant, as they are the same on the two platforms. In this case, only the participation price at the two platforms matters, while expectations on developer participation cancel out. Thus, in this case, consumer demand is as in a classical Hotelling setting as expectations on developer participation on either platform do not matter. A direct consequence of this is that we are back to the result as in the traditional models without network effects (Thisse and Vives, 1988).

[27] Our results are robust to the case of endogenous multi-homing by developers. To show this, we assume developers are distributed on a Hotelling line as in Liu and Serfes (2013) and they can choose to multi-home or single-home across the platforms as in Belleflamme and Peitz (2019a). Under this market structure, our main results hold. The reason for this is because in this case, developer demand behavior is similar to the main model.

6 Comparison with Liu and Serfes (2013)

To isolate the key drivers of our results, we consider three extensions that help clarify the reason for differences in our results with respect to Liu and Serfes (2013).

6.1 Developer-side private personalized fees. In this case, the platforms employ private personalized fees for developers, allowing for a comparison with the case of private personalized pricing discussed in Liu and Serfes (2013, Section 3.5).[28] The demand of developers in each platform is larger than that in Section 4.2 due to personalized fees for developers. Except for this demand expansion and the complete exploitation of developer surplus, the competitive environment is similar to Section 4.2. Therefore, personalized pricing benefits platforms if ϕ is larger than a threshold value.

6.2 Developer-side private uniform fees. In this case, demand of developers is elastic, and as developers fees are private, consumers do not observe the fees charged to developers.[29] As a result, these (developer) fees do not influence consumers' expectations on the mass of developers in both uniform and personalized pricing regimes. This implies that the equilibrium fees and prices in the two regimes are independent of θ and are identical to those in the main model without the interaction value of consumers θ (substituting $\theta = 0$ into the results in the main model). As the parameter of interest for our main result is not the interaction value of consumers but that of developers ϕ, the comparison is qualitatively similar to that in the main model. Intuitively, private fees to developers make demands less responsive in both regimes. Nevertheless, the same trade-off between competition dampening and competition enhancing effect of personalized pricing continue to hold.

6.3 Consumer-side observable personalized prices. In this case, we allow consumer-side personalized prices to be observable. This observability induces fierce competition

[28] A detailed analysis of this case is available upon request.

[29] A detailed analysis of this case is available upon request.

between platforms that can credibly offer negative personalized prices to some consumers so as to attract developers and recoup profits on the developer side. In this case, our results do not hold as the competition-dampening effect of (private) personalized prices is absent in this setting.

The above comparisons suggest that the combination of elastic developer demand and private personalized prices is a key feature for our results to hold. We summarize these results in Liu and Serfes (2013) (hereafter, LS) and the ones verified by us in Table 1.

Developer side conditions		Types of personalized pricing	
Fee schedules	Demand system	Observable	Private
Observable uniform fee	Hotelling	—	Our (5.2) Y
Observable personalized fees	Hotelling	LS (§3.3) Y	—
Private personalized fees	Hotelling	—	LS (§3.5) N
Observable uniform fee	elastic demand	Our (6.3) N	Our (main) Y
Observable commission fee	elastic demand	—	Our (S.1) Y
Private uniform fee	elastic demand	—	Our (6.2) Y
Private personalized fees	elastic demand	—	Our (6.1) Y

Table 1: Comparison between Liu and Serfes (2013) and ours

Note: Personalized fees are used if and only if platforms offer personalized prices.

The columns in the table focus on three aspects of the developer side. The first column refers to variations in fee schedules for developers (observable or private, uniform or personalized). The second column indicates whether developer demand is modeled as elastic or inelastic (Hotelling) demand. The third and fourth columns indicate whether the personalized pricing regime involves observable or private personalized pricing. In these columns, (LS) implies that this case has been studied in Liu and Serfes (2013), and (Y) or (N) refers to whether our results hold or not in these extensions. Finally, the entry (Our) implies that we have done this extension, which is absent in Liu and Serfes (2013).

7 Policy Implications

In this section, we focus on the policy implications arising from our research and key takeaways for policymakers. Recently, the economic effects of personalized/targeted pricing have become an important issue. For instance, The Federal Trade Commission (FTC) opened an inquiry into the potential impact of this pricing practice on competition and consumers using these platforms.[30] We provide some insights into how personalized pricing affects markets and the impact of its prohibition. Personalized pricing, particularly the prohibition of first-degree price discrimination, has been proposed as a policy tool to mitigate scenarios where the rent *appropriation effect* outweighs the *demand expansion effect* (Bourreau and De Streel, 2018).

Policy Implication 1 *Under competition in two-sided markets, personalized prices on the consumers' side benefit developers. Thus, any regulation that bans personalized prices to consumers hurts developers.*

In our study, we uncover intriguing effects of altering the nature of pricing on the consumers' and developers' (complementors') sides. Specifically, the ability to set personalized prices for consumers incentivizes the platform to increase gross consumer surplus by expanding participation on the developers' side. In simpler terms, the platform's capacity to (unilaterally) extract surplus from consumers more effectively under personalized pricing serves as motivation to boost developer participation. This dynamic, where price discrimination on one side influences the participation of the other side, is reminiscent of the findings in De Cornière et al. (2025). Increased competition for consumers prompts the rival platform to respond by lowering developer fees, ultimately benefiting developers in equilibrium. Policymakers, when contemplating the prohibition of personalized prices on the consumers' side, must also consider the potential (negative) impact of such a regulation on the developers' side.

Policy Implication 2 *Greater transparency on personalized pricing to complementors may be a more effective tool than an outright ban on personalized pricing.*

In recent policy reports, there have been discussions regarding the mandate of greater transparency for consumers regarding the algorithms employed by platforms for personalized pricing (see Bourreau and De Streel, 2018, Rott et al., 2022). The focus of these

[30] See the FTC inquiry on personalized pricing on 23rd July 2024 link.

regulations is to maintain consumer trust in the market and avoid market failures. While the above Policy Implication 1 advocates for transparency, the transparency of consumer prices is directed toward the developers' side and is nuanced. Specifically, the above policy implication suggests that personalized pricing algorithms should be made more transparent to developers (complementors).[31] Under personalized pricing, developers (complementors) do not observe consumer prices and, therefore, cannot base their expectations on them (in contrast to the uniform pricing case). This absence of information makes consumer prices less sensitive to the network value of developers. The competition-dampening effect of a lack of information can be avoided by informing developers about the algorithm employed to implement personalized prices. Thus, this policy retains the competitive benefits of personalized pricing without imposing any restrictions on platform strategies.

Policy Implication 3 *In markets characterized by dominant incumbent platforms, prohibiting price discrimination may encourage entry and increase competition.*

This policy implication is directly derived from Corollary 1 and Proposition 4. In markets with a very dominant incumbent (high h), personalized pricing only increases their profitability, further entrenching their market power. This entrenchment arises at the expense of the profitability of smaller competing platforms. Consequently, (smaller) platforms that may be interested in entering the market may be dissuaded when the dominant platform employs personalized pricing. Focusing on consumer surplus, we find that as the incumbent gets more dominant (as h increases), the consumer surplus under personalized pricing falls in relation to uniform prices. This can be observed directly by the fact that the difference in price between the two regimes increases (see discussion after Proposition 4) as its market power increases (as h increases). This implication informs directly the inquiry of the Federal Trade Commission (FTC) on the impact of personalized pricing on competition and consumers.

[31] While pricing algorithms are proprietary and complex, greater transparency can be implemented in various ways without sharing the intellectual property: (i) share high level logic of their pricing algorithm, (ii) share weights places on different attributes, (iii) offer a breakdown on how the price was arrived at.

However, we would like to highlight that these policy suggestions must be implemented keeping in mind the policy goals of the regulators, such as enhancing developer surplus or increasing competitive intensity in the platform market.

8 Conclusions

We revisited the issue of personalized pricing under competition in two-sided markets with consumers and developers and its impact on welfare in markets featuring network effects. Contrary to the established result that consumers benefit when competing platforms employ personalized prices, we showed a contrasting outcome: personalized pricing harms consumer welfare but improves total surplus when the developers' network interaction value is sufficiently high. This intriguing result, driven solely by network effects, provides a rationale for why platforms like Uber, among others, find it profitable to implement personalized pricing in platform markets. Specifically, in the presence of network effects, publicly observable consumer prices serve as a tool to shape favorable expectations regarding network interaction value at the platform. Consequently, as the value of network interactions increases, platforms fiercely compete to attract consumers. The implementation of personalized pricing, which makes consumer prices private, dampens this competitive channel due to the presence of network effects. However, when the value of network interactions is high, the competition-dampening effect of personalized prices dominates, making competing platforms better off. These results enable us to derive valuable policy implications and inform regulators.

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Appendix

Proof of Lemma 1. Solving simultaneously the system first order conditions in equations (6) and (7) yields the symmetric equilibrium fees as

$$p_i^U = t - \frac{\phi(3\theta + \phi)}{4}, \quad l_i^U = \frac{\phi - \theta}{4} \text{ for } i = 1, 2.$$

Substituting these prices into the profit expression yields the expressions presented in Lemma 1.

$$\begin{aligned}
\pi_i^U &= \frac{t}{2} - \frac{\theta^2 + 6\theta\phi + \phi^2}{16}, \quad N_i^U = \frac{1}{2}, \quad D_i^U = \frac{\theta + \phi}{4}, \\
CS^U &= \int_0^{N_1^U} (w + \theta D_1^U - tx - p_1^U)dx + \int_{N_1^U}^1 (w + \theta D_2^U - t(1-x) - p_2^U)dx \\
&= w - \frac{5t}{4} + \frac{\theta^2 + 4\theta\phi + \phi^2}{4}, \\
PS^U &= \int_0^{D_1^U} (\phi N_1^U - k - l_1^U)dk + \int_0^{D_2^U} (\phi N_2^U - k - l_2^U)dk = \frac{(\theta + \phi)^2}{16}.
\end{aligned}$$

■

Proof of Lemma 2. Solving simultaneously the system first order conditions in equations (15) yields the symmetric equilibrium fees as

$$l_1^P = l_2^P = \frac{t(\phi-\theta)-\theta\phi^2}{4t-3\theta\phi}.$$

Substitute the developer fees into personalized consumer prices and obtain

$$p_1^P(x) = \begin{cases} t(1-2x) & \text{if } x \leq 1/2, \\ 0 & \text{if } x > 1/2, \end{cases} \quad p_2^P(x) = \begin{cases} 0 & \text{if } x \leq 1/2, \\ t(2x-1), & \text{if } x > 1/2, \end{cases}$$

Substituting these prices into the profit expression yields the expressions presented in Lemma 2.

$$\begin{aligned}
\Pi_i^P &= \frac{t}{4} - \frac{((\theta-\phi)t+\theta\phi^2)(2(\theta+\phi)t-\theta\phi^2)}{2(4t-3\theta\phi)^2}, \quad N_i^P = \frac{1}{2}, \quad D_i^P = \frac{2(\theta+\phi)t-\theta\phi^2}{2(4t-3\theta\phi)}, \\
CS^P &= \int_0^{\bar{x}^P} (w+\theta D_1^P - tx - p_1^P(x))dx + \int_{\bar{x}^P}^1 (w+\theta D_2^P - t(1-x) - p_2^P(x))dx \\
&= w - \frac{12t^2-\theta(4\theta+13\phi)t+2\theta^2\phi^2}{4(4t-3\theta\phi)}, \\
PS^P &= \int_0^{D_1^P} (\phi N_1^P - k - l_1^P)dk + \int_0^{D_2^P} (\phi N_2^P - k - l_2^P)dk = \frac{(2(\theta+\phi)t-\theta\phi^2)^2}{4(4t-3\theta\phi)^2}.
\end{aligned}$$

■

Proof of Proposition 1. The average consumer price under personalized pricing is given as

$$\mathbb{E}p^P = \int_0^{N_1^P} p_1^P(x)dx + \int_{N_1^P}^1 p_2^P(x)dx = \frac{t}{2}.$$

Comparing the above-average (personalized) price with the consumer price in the uniform pricing case yields

$$\mathbb{E}p^P - p^U = \frac{\phi(3\theta+\phi)}{4} - \frac{t}{2}.$$

The above expression is positive when $\phi > \widehat{\phi} = (\sqrt{8t+9\theta^2}-3\theta)/2$.

Next, comparing the developer fees in the two regimes yields

$$l_i^P - l_i^U = -\frac{\theta\phi(3\theta+\phi)}{4(4t-3\theta\phi)} < 0.$$

The above inequality always holds under Assumption 1.

■

Proof of Proposition 2. The proof is straightforward by just reviewing the equation (18) and (19). ■

Proof of Proposition 3. Comparing the consumer surplus under personalized pricing with the consumer surplus under uniform pricing yields

$$\Delta CS = CS^P - CS^U = \frac{8t^2 + \theta\phi(3\theta+\phi)(\theta+3\phi) - 2t\phi(9\theta+2\phi)}{4(4t-3\theta\phi)}.$$

The sign of the above expression depends on the numerator, which we define as $\Lambda := 8t^2 + \theta\phi(3\theta+\phi)(\theta+3\phi) - 2t\phi(9\theta+2\phi)$.

Equating Λ and solving with 0 and solving with respect to ϕ yields

$$\phi_1 = \frac{\left(4t + \epsilon^{1/3} - 10\theta^2 + \frac{16t^2+73\theta^4+82\theta^2 t}{\epsilon^{1/3}}\right)}{9\theta}.$$

where $\epsilon = -595\theta^6 + 64t^3 - 480\theta^2 t^2 - 1392\theta^4 t$
$+ 9\sqrt{6}\sqrt{-72\theta^{12} - 256\theta^2 t^5 - 672\theta^4 t^4 + 276\theta^6 t^3 + 1606\theta^8 t^2 + 711\theta^{10} t}$.

Comparing the profit under personalized pricing with the profit under uniform pricing yields

$$\Delta\Pi = \Pi_i^P - \Pi_i^U = \frac{32t^2\theta(6\theta+\phi) + \theta^2\phi^2(3\theta+\phi)(3\theta+17\phi) - 4t\theta\phi(6\theta^2+12\phi^2+47\theta\phi) - 64t^3}{16(4t-3\theta\phi)^2}.$$

The sign of the above expression depends on the numerator which we define as $\Gamma := 32t^2\theta(6\theta+\phi) + \theta^2\phi^2(3\theta+\phi)(3\theta+17\phi) - 4t\theta\phi(6\theta^2+12\phi^2+47\theta\phi) - 64t^3$.

Equating Γ and solving with 0 and solving with respect to ϕ yields ϕ_2. We suppress the expression for ϕ_2 for brevity. It is available upon request. Simulate the relevant parameter range and comparing ϕ_1 and ϕ_2, we note that $\phi_1 < \phi_2$. ■

Proof of Proposition 4. In the asymmetric case, the utility when a consumer purchases from platform 1 is $U_1(p_1, D_1^e, x) = w + h + \theta D_1^e - p_1 - tx$ $(h > 0)$.

Assumption 2 *The exogenous parameters, t, θ, and ϕ, satisfy $t > \underline{t}$ and $0 < h < \bar{h}$, where*

$$\underline{t} \equiv \max\left\{\frac{\theta^2 + 4\theta\phi + \phi^2 + 2h}{6}, \frac{4h + 2\theta^2 + 5\theta\phi + \sqrt{4(2h+\theta^2)^2 + 9\theta^2\phi^2 - 4\theta\phi(5\theta^2 - 2h)}}{8}\right\},$$

$$\bar{h} \equiv \frac{(4t^2+\theta^2\phi^2-t\theta(2\theta+5\phi))(2t(4-\theta-\phi)-\theta\phi(6-\phi))}{(4t-3\theta\phi)(2t(\theta+\phi)-\theta\phi^2)}.$$

These conditions ensure that the second order conditions are satisfied and the demands are within the bounds of the distributions.

Uniform pricing regime. Following the same method in the benchmark model, the mass of consumers and developers are

$$N_1^\star(\cdot) = \frac{1}{2} - \frac{(p_1-p_2)+\theta(l_1-l_2)-h}{2(t-\theta\phi)}, D_1{}^\star(\cdot) = \frac{\phi(\theta(l_1+l_2)-p_1+p_2+t+h)-\theta\phi^2-2l_1t}{2(t-\theta\phi)},$$
$$N_2{}^\star(\cdot) = \frac{1}{2} - \frac{(p_2-p_1)+\theta(l_2-l_1)+h}{2(t-\theta\phi)}, D_2{}^\star(\cdot) = \frac{\phi(\theta(l_1+l_2)+p_1-p_2+t-h)-\theta\phi^2-2l_2t}{2(t-\theta\phi)}.$$

The profit functions of platforms are

$$\Pi_1^\star(p_1,p_2,l_1,l_2) \;= p_1N_1{}^\star(\cdot)+l_1D_1^\star(\cdot),\ \Pi_2^\star(p_1,p_2,l_1,l_2).$$

Checking the first-order condition, we derive the equilibrium fees.

$$p_1^U = t - \frac{\phi(3\theta+\phi)}{4} + \frac{(4t-\phi(3\theta+\phi))h}{2\left(6t-\theta^2-4\theta\phi-\phi^2\right)}, p_2^U = t - \frac{\phi(3\theta+\phi)}{4} - \frac{(4t-\phi(3\theta+\phi))h}{2\left(6t-\theta^2-4\theta\phi-\phi^2\right)},$$
$$l_1^U = \frac{1}{4}\left(\phi-\theta+\frac{2(\phi-\theta)h}{6t-\theta^2-4\theta\phi-\phi^2}\right),\ l_2^U = \frac{1}{4}\left(\phi-\theta-\frac{2(\phi-\theta)h}{6t-\theta^2-4\theta\phi-\phi^2}\right).$$

The profit of each platform is

$$\Pi_1^U = \frac{\left(8t-(\theta^2+6\theta\phi+\phi^2)\right)\left(6t-(\theta^2+4\theta\phi+\phi^2)+2h\right)^2}{16\left(6t-(\theta^2+4\theta\phi+\phi^2)\right)^2},$$
$$\Pi_2^U = \frac{\left(8t-(\theta^2+6\theta\phi+\phi^2)\right)\left(6t-(\theta^2+4\theta\phi+\phi^2)-2h\right)^2}{16\left(6t-(\theta^2+4\theta\phi+\phi^2)\right)^2}.$$

Consumer, developer, and total surplus are

$$CS^U = w - \frac{5t}{4} + \frac{1}{4}\left((\theta^2+4\theta\phi+\phi^2)+2h+\frac{4th^2}{\left(6t-\theta^2-4\theta\phi-\phi^2\right)^2}\right),$$
$$PS^U = \frac{1}{16}(\theta+\phi)^2\left(1+\frac{4h^2}{\left(6t-\theta^2-4\theta\phi-\phi^2\right)^2}\right),$$
$$TS^U = w - \frac{t}{4} + \frac{1}{16}\left(3(\theta+\phi)^2+8h+\frac{\left(80t-4\left(\theta^2+10\theta\phi+\phi^2\right)\right)h^2}{\left(6t-\theta^2-4\theta\phi-\phi^2\right)^2}\right).$$

Personalized Pricing regime. In this case, the personalized consumer fee of each platform is:

$$p_1(x) = \begin{cases} h + \theta(D_1^e - D_2^e) + t(1-2x) & \text{if } x \le \bar{x}, \\ 0 & \text{if } x > \bar{x}, \end{cases} \tag{20}$$

$$p_2(x) = \begin{cases} 0 & \text{if } x \le \bar{x}, \\ -h + \theta(D_2^e - D_1^e) + t(2x-1), & \text{if } x > \bar{x}, \end{cases} \tag{21}$$

where $\bar{x} = (h + t + \theta(D_1^e - D_2^e))/(2t)$. Substituting the prices into developers' demand, we derive the mass of developers as below.

$$D_1^{\star\star}(l_1, l_2) = \frac{\phi(h + \theta(l_1 + l_2) + t) - \theta\phi^2 - 2l_1 t}{2(t - \theta\phi)}, \; D_2^{\star\star}(l_1, l_2) = \frac{\phi(-h + \theta(l_1 + l_2) + t) - \theta\phi^2 - 2l_2 t}{2(t - \theta\phi)}.$$

The profit functions of platforms are

$$\begin{aligned} \Pi_1(l_1, l_2) &= \int_0^{x^{\star\star}} p_1(x)dx + l_1 D_1^{\star\star}(\cdot) \\ &= \frac{t(t - \theta\phi + h + \theta(l_2 - l_1))^2}{4(t-\theta\phi)^2} + \frac{(\phi(h + t - \theta\phi + \theta l_2) - (2t - \theta\phi)l_1)) l_1}{2(t - \theta\phi)}, \\ \Pi_2(l_1, l_2) &= \int_{x^{\star\star}}^1 p_2(x)dx + l_2 D_2^{\star\star}(\cdot) \\ &= \frac{t(t - \theta\phi - h + \theta(l_1 - l_2))^2}{4(t-\theta\phi)^2} + \frac{(\phi(t - \theta\phi - h + \theta l_2) - (2t - \theta\phi)l_2)) l_2}{2(t - \theta\phi)}, \end{aligned}$$

where $x^{\star\star} = (h + t + \theta(D_1^{\star\star} - D_2^{\star\star}))/(2t)$. Checking the first-order condition, we derive the equilibrium developer fees and $x^{\star\star}$:

$$\begin{aligned} l_1^P &= \frac{t(\phi - \theta) - \theta\phi^2}{4t - 3\theta\phi} + \frac{(t(\phi - \theta) - \theta\phi^2)h}{4t^2 - \theta(2\theta + 5\phi)t + \theta^2\phi^2}, \\ l_2^P &= \frac{t(\phi - \theta) - \theta\phi^2}{4t - 3\theta\phi} - \frac{(t(\phi - \theta) - \theta\phi^2)h}{4t^2 - \theta(2\theta + 5\phi)t + \theta^2\phi^2}, \\ \bar{x}^{\star\star} &= \frac{1}{2} + \frac{(4t - 3\theta\phi)h}{2(4t^2 - \theta(2\theta + 5\phi)t + \theta^2\phi^2)}. \end{aligned}$$

Then, we substitute the developer fees into personalized consumer fees and obtain

$$p_1^P(x) = \begin{cases} t(1-2x) + \dfrac{(4t - 3\theta\phi)th}{4t^2 - \theta(2\theta + 5\phi)t + \theta^2\phi^2} & \text{if } x \le \bar{x}^{\star\star}, \\ 0 & \text{if } x > \bar{x}^{\star\star}, \end{cases}$$

$$p_2^P(x) = \begin{cases} 0 & \text{if } x \le \bar{x}^{\star\star}, \\ t(2x-1) - \dfrac{(4t - 3\theta\phi)th}{4t^2 - \theta(2\theta + 5\phi)t + \theta^2\phi^2}, & \text{if } x > \bar{x}^{\star\star}. \end{cases}$$

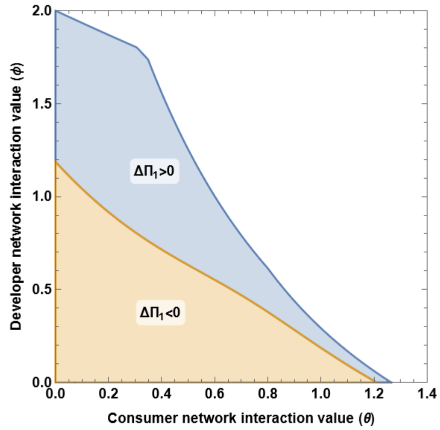
The profit of each platform is

$$
\begin{aligned}
\Pi_1^P &= \frac{(4t+\phi^2)\left(2\theta^2\phi^2+4t^2-\theta t(\theta+6\phi)\right)\left(\theta^2\phi^2+h(4t-3\theta\phi)+4t^2-\theta t(2\theta+5\phi)\right)^2}{4(4t-3\theta\phi)^2\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2},\\
\Pi_2^P &= \frac{(4t+\phi^2)\left(2\theta^2\phi^2+4t^2-\theta t(\theta+6\phi)\right)\left(\theta^2\phi^2+h(3\theta\phi-4t)+4t^2-\theta t(2\theta+5\phi)\right)^2}{4(4t-3\theta\phi)^2\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2}.
\end{aligned}
$$

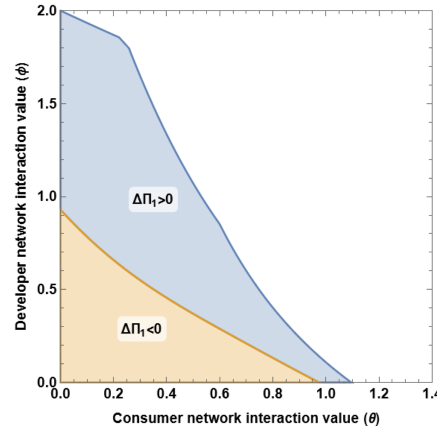
Consumer, developer, and total surplus are

$$
\begin{aligned}
CS^P &= w-\frac{12t^2-\theta(4\theta+13\phi)t+2\theta^2\phi^2}{4(4t-3\theta\phi)}+\frac{h}{2}-\frac{(4t-3\theta\phi)^2th^2}{4\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2},\\
PS^P &= \frac{\left(2(\theta+\phi)t-\theta\phi^2\right)^2}{4}\left(\frac{1}{(4t-3\theta\phi)^2}+\frac{h^2}{\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2}\right),\\
TS^P &= w-\frac{16t^3-12(\theta^2+4\theta\phi+\phi^2)t^2+\theta\phi(12\theta^2+37\theta\phi+16\phi^2)t-\theta^2\phi^3(6\theta+5\phi)}{4(4t-3\theta\phi)^2}\\
&\quad+\frac{h}{2}+\frac{(16t^3-4(\theta^2+4\theta\phi-3\phi^2)t^2+\theta\phi^2(\theta^2-16\phi)t+5\theta^2\phi^4)h^2}{4\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2}.
\end{aligned}
$$

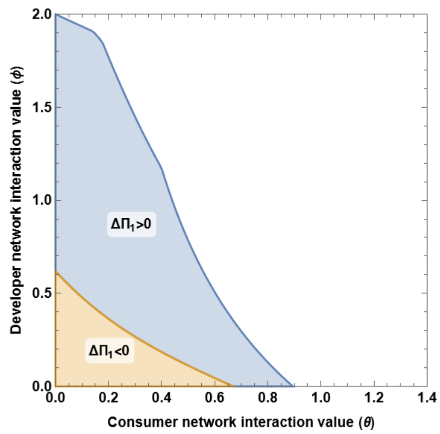
Profit comparison. Let's denote the profit differences as $\Delta\Pi_1 = \Pi_1^P - \Pi_1^U$ and $\Delta\Pi_2 = \Pi_2^P - \Pi_2^U$. Figures (5) and (6) plot these differences as h changes.



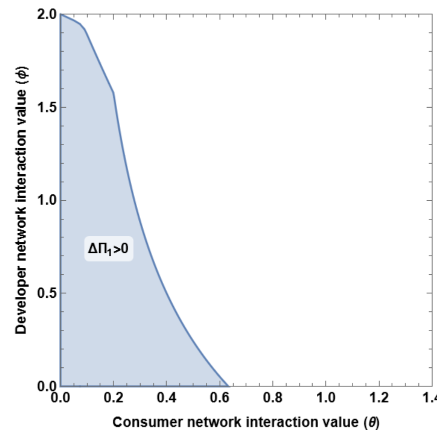
(a) $h = 0.2$



(b) $h = 0.4$

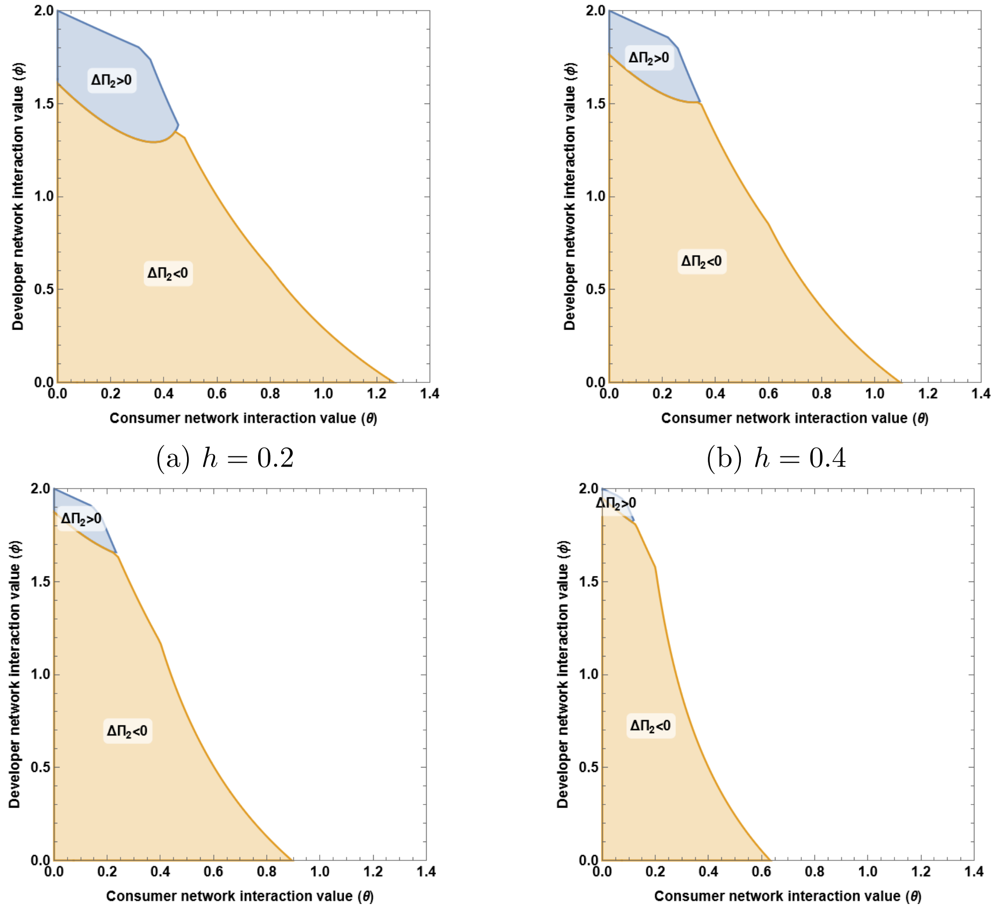


(c) $h = 0.6$



(d) $h = 0.8$

Figure 5: Profitability of platform 1 as h changes for $t = 1$.



(a) $h = 0.2$ (b) $h = 0.4$

(c) $h = 0.6$ (d) $h = 0.8$

Figure 6: Profitability of platform 2 as h changes for $t = 1$.

Consumer surplus comparison. Comparing the consumer surplus under personalized pricing with that under uniform pricing yields

$$\begin{aligned}
\Delta CS &= CS^P - CS^U \\
&= -\frac{(3\theta+\phi)(\theta+3\phi)}{12} + \frac{7t}{18} + \frac{4t+9\theta^2}{9(4t-3\theta\phi)} \\
&\quad - \left(\frac{(4t-3\theta\phi)^2}{4\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2} + \frac{1}{\left(6t-\theta^2-4\theta\phi-\phi^2\right)^2}\right) th^2.
\end{aligned}$$

This difference is always decreasing in h as

$$\frac{\partial \Delta CS}{\partial h} = -2\left(\frac{(4t-3\theta\phi)^2}{4\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2} + \frac{1}{\left(6t-\theta^2-4\theta\phi-\phi^2\right)^2}\right)t < 0.$$

Total surplus comparison. Comparing the total surplus under personalized pricing with that under uniform pricing yields

$$\begin{aligned}\Delta TS &= TS^P - TS^U \\ &= \frac{Hh^2}{4\left(6t-\theta^2-4\theta\phi-\phi^2\right)^2\left(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2\right)^2} \\ &\quad + \frac{\theta\phi(3\theta+\phi)\left(8(\theta+\phi)t-\theta\phi(9\theta+7\phi)\right)}{16(4t-3\theta\phi)^2},\end{aligned}$$

where $H \equiv 256t^5 - 128\phi(3\theta-2\phi)t^4 - 8(4\theta^4+11\theta^3\phi+10\theta^2\phi^2+109\theta\phi^3+16\phi^4)t^3 + \phi(12\theta^5 + 117\theta^4\phi + 470\theta^3\phi^2 + 1053\theta^2\phi^3 + 272\theta\phi^4 + 12\phi^5)t^2 - \theta\phi^2(3\theta^5 + 58\theta^4\phi + 294\theta^3\phi^2 + 530\theta^2\phi^3 + 187\theta\phi^4 + 16\phi^5)t + \theta^2\phi^4(3\theta+\phi)(2\theta^3+16\theta^2\phi+25\theta\phi^2+5\phi^3)$. ΔTS is increasing in h if and only if $H > 0$.

We denote the threshold of ϕ as ϕ_3, then ΔTS is increasing in h if and only if $\phi < \phi_3$. In Figure 7, we show that ϕ_3 is higher than ϕ_2, where ϕ_2 is the threshold of ϕ under which personalized pricing improves platforms' profits in the symmetric case.

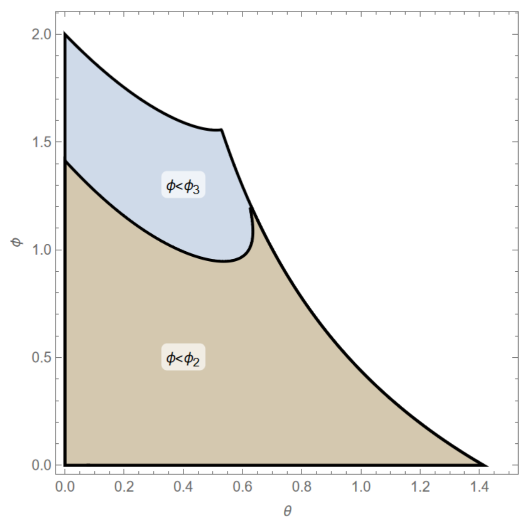


Figure 7: Comparison of ϕ_2 and ϕ_3.

■

Online Appendix

Welfare implications of personalized pricing in competitive platform markets: The role of network effects

Qiuyu Lu* Noriaki Matsushima† Shiva Shekhar‡

June 30, 2025

S.1 Commission fees for developers

We show a qualitatively similar result under the case in which the platforms impose commission fees on developers. We change the payoffs of developers and platforms.

The payoff of a developer of type k interacting with consumers at platform $i \in 1, 2$ is

$$\pi_i^{Dev}(r_i, N_i^e) - k = (1 - r_i)\phi N_i^e - k,$$

where r_i is the commission rate of platform i. We interpret ϕ as the per-consumer revenue that a developer earns from a transaction with a consumer.

Platform i obtains $r_i\phi$ from one transaction between a developer and a consumer. The profit of platform i is $\Pi_i = p_i N_i + r_i \phi N_i D_i$. The following assumptions are imposed to ensure the second-order conditions are fulfilled and demands within the distributional bounds.

Assumption 1. *The exogenous parameters, t, θ, and ϕ, satisfy $\theta + \phi < 4$ and $t > \underline{t} \equiv \max\{\theta^2 + \theta\phi,\ \frac{4\theta}{4-\theta-\phi},\ \frac{(3\theta+\phi)(\theta+\phi)}{8}\}$.*

Uniform pricing The masses of consumers at platforms 1 and 2 are $N_i(D_1^e, D_2^e, p_1, p_2)$ in (3) $(i = 1, 2)$, as the same as those in the main model. The mass of developers active on platform i is

$$D_i(N_i^e, r_i) = k_i(N_i^e, r_i) = (1 - r_i)\phi N_i^e.$$

Using the four equations, we obtain demands as a function of prices and fees:

$$\begin{aligned}
N_i^*(p_i, p_{-i}, r_i, r_{-i}) &= \frac{1}{2} + \frac{\theta\phi(r_j - r_i) + 2(p_j - p_i)}{2(2t - (2 - r_1 - r_2)\theta\phi)},\\
D_i^*(p_i, p_{-i}, r_i, r_{-i}) &= \frac{\phi(1 - r_i)}{2}\left(1 + \frac{\theta\phi(r_j - r_i) + 2(p_j - p_i)}{2t - (2 - r_1 - r_2)\theta\phi}\right).
\end{aligned}$$

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The profit of platform i is $p_i N_i^*(p_i, p_{-i}, r_i, r_{-i}) + r_i\phi N_i^*(p_i, p_{-i}, r_i, r_{-i}) D_i^*(p_i, p_{-i}, r_i, r_{-i})$. Solving the first-order conditions leads to

$$p_i^U = t - \frac{(\theta+\phi)^2}{4}, \quad r_i^U = \frac{\phi-\theta}{2\phi}.$$

The profits of the platforms are

$$\Pi_i^U = \frac{t}{2} - \frac{(\theta+\phi)(3\theta+\phi)}{16}.$$

The consumer surplus is

$$CS^U = w - \frac{5t}{4} + \frac{(\theta+\phi)(2\theta+\phi)}{4}.$$

The developer surplus is

$$PS^U = \frac{(\theta+\phi)^2}{16}.$$

The total surplus is given as

$$TS^U = \sum_{i=1}^{2} \Pi_i^U + CS^U + PS^U = w - \frac{t}{4} + \frac{3(\theta+\phi)^2}{16}.$$

Personalized pricing The price schedules of platforms 1 and 2 are the same as in (9) and (10). The masses of consumers at platforms 1 and 2 are also the same as in the main model. We obtain the mass of developers active on platform i is

$$D_i^{**}(r_i, r_{-i}) = \frac{(1-r_i)\phi}{2}\left(1 + \frac{\theta\phi(r_j - r_i)}{2t - (2 - r_1 - r_2)\theta\phi}\right).$$

Using this outcome, we obtain the price schedules of platforms 1 and 2:

$$\begin{aligned}
p_1^{**}(r_1, r_2, x) &= \begin{cases} t(1-2x) + \dfrac{\theta\phi(r_2 - r_1)t}{2t - \theta\phi(2 - r_1 - r_2)} & \text{if } x \le \bar{x}^{**}, \\ 0 & \text{if } x > \bar{x}^{**}, \end{cases} \\
p_2^{**}(r_1, r_2, x) &= \begin{cases} t(2x-1) + \dfrac{\theta\phi(r_1 - r_2)t}{2t - \theta\phi(2 - r_1 - r_2)} & \text{if } x \ge \bar{x}^{**}, \\ 0 & \text{if } x < \bar{x}^{**}, \end{cases} \\
&\text{where } \bar{x}^{**} = N_1^{**}(r_1, r_2) = \frac{1}{2}\left(1 + \frac{\theta\phi(r_2 - r_1)}{2t - (2 - r_1 - r_2)\theta\phi}\right).
\end{aligned}$$

$N_2^{**}(r_1, r_2) = 1 - N_1^{**}(r_1, r_2)$. The profits of platforms 1 and 2 are

$$\int_0^{\bar{x}^{**}} p_1^{**}(r_1, r_2, x)dx + r_1\phi N_1^{**}(r_1, r_2) D_1^{**}(r_1, r_2),$$
$$\int_{\bar{x}^{**}}^{1} p_2^{**}(r_1, r_2, x)dx + r_2\phi N_2^{**}(r_1, r_2) D_2^{**}(r_1, r_2).$$

Solving the first-order conditions leads to

$$r_i^P = \frac{-(t-\theta\phi) + \sqrt{t(t - \theta(\theta+\phi))}}{\theta\phi}.$$

The profits of the platforms are (note that ${\Pi_i^P}_{\theta\to 0+} = t/4 + \phi^2/16$):

$$\Pi_i^P = \frac{-2t(t-\theta(\theta+\phi)) + (2t-\theta\phi)\sqrt{t(t-\theta(\theta+\phi))}}{4\theta^2}.$$

The consumer surplus is

$$CS^P = w - \frac{t}{4} - \frac{\sqrt{t(t-\theta(\theta+\phi))}}{2}.$$

The developer surplus is (note that ${PS^P}_{\theta\to 0+} = \phi^2/16$):

$$PS^P = \frac{(t-\sqrt{t(t-\theta(\theta+\phi))})^2}{4\theta^2}.$$

The total surplus is given as

$$\begin{aligned} TS^P &= \sum_{i=1}^{2}\Pi_i^P + CS^P + PS^P, \qquad \text{(B.1)} \\ &= w + \frac{3t\theta\phi - 2\theta(\theta+\phi)\sqrt{t(t-\theta(\theta+\phi))} + 2t(\theta^2+\sqrt{t(t-\theta(\theta+\phi))}) - 2t^2}{4\theta^2}. \end{aligned}$$

Welfare effects of pricing regimes

We start by comparing prices. The average consumer price in personalized pricing is $t/2$ because of the symmetry of the platforms. Calculating $t/2 - p_i^U$ and $r_i^P - r_i^U$ leads to the following observation, which is qualitatively similar to Proposition 1: **Observation:** *The average consumer price in personalized pricing is higher than the price in uniform pricing when $\phi \geq \widehat{\phi}_A := \sqrt{2t} - \theta$. Commission rates are unambiguously lower in personalized pricing than in uniform pricing — i.e., $r_i^P < r_i^U$.*

Denote the difference in developer surplus and total surplus respectively as $\Delta PS = PS^P - PS^U$ and $\Delta TS = TS^P - TS^U$, we obtain the following outcomes, which are summarized as in Proposition 2:

$$\begin{aligned} \Delta PS &= \frac{8t(t-\sqrt{t(t-\theta(\theta+\phi))}) - \theta(\theta+\phi)(4t+\theta(\theta+\phi))}{16\theta^2} > 0, \qquad \text{(B.2)} \\ \Delta TS &= \frac{8\sqrt{t(t-\theta(\theta+\phi))}(t-\theta(\theta+\phi)) + 3\theta(\theta+\phi)(4t-\theta(\theta+\phi)) - 8t^2}{16\theta^2} > 0. \qquad \text{(B.3)} \end{aligned}$$

The above relations hold under Assumption 3. To elaborate, we can show that the denominator of ΔTS is zero at $\theta = 0$. Further, we find that ΔTS is increasing in $\theta(> 0)$. The first-order derivative of this difference with respect to θ is

$$\frac{-2t(t-\theta(\theta+\phi))(4t+\theta(2\theta-\phi)) + (2t(4t-3\theta\phi) - 3\theta^3(\theta+\phi))\sqrt{t(t-\theta(\theta+\phi))}}{8\theta^3\sqrt{t(t-\theta(\theta+\phi))}},$$

which is non-negative for any $t > 0$, $0 < \theta < 2$, and $0 < \phi < 2$.

Next, we examine the effects of personalized pricing on profits and consumer surplus. To do so, we define the difference in profits and consumer surplus between the two regimes as $\Delta\Pi = \Pi_i^P - \Pi_i^U = \frac{4((2t-\theta\phi)\sqrt{t(t-\theta(\theta+\phi))} - 2t(t-\theta\phi)) + \theta^2(\theta+\phi)(3\theta+\phi)}{16\theta^2}$ and $\Delta CS = CS^P - CS^U = t - \frac{(2\theta+\phi)(\theta+\phi)}{4} - \frac{\sqrt{t(t-\theta(\theta+\phi))}}{2}$. Furthermore, we obtain the following proposition, which is qualitatively similar

to Proposition 3:
The following relationship holds.

- When $0 < \phi < \phi_{1a}$, consumer surplus (platform profit) under personalized pricing is higher (lower) higher than under uniform pricing — i.e., $\Delta CS > 0$ and $\Delta\Pi < 0$.
- When $\phi_{1a} < \phi < \phi_{2a}$, consumer surplus and platform profits under personalized pricing are lower than under uniform pricing — i.e., $\Delta CS < 0$ and $\Delta\Pi < 0$.
- When $\phi_{2a} < \phi$, consumer surplus (platform profit) under personalized pricing is lower (higher) than under uniform pricing — i.e., $\Delta CS < 0$ and $\Delta\Pi > 0$.

The value of ϕ_{1a} and ϕ_{2a} are not analytically tractable. Nevertheless, as in the benchmark, we provide an analogous Figure below to the Figure (1).

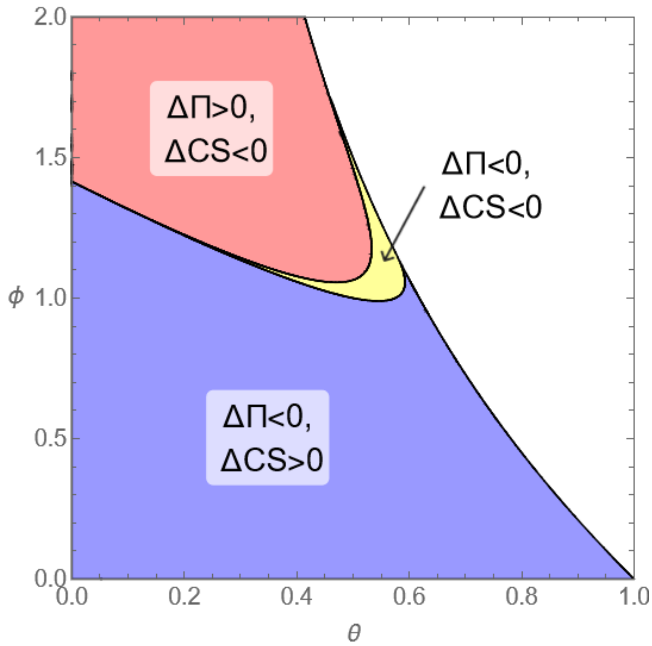


Figure 1: Comparison of profits and consumer surplus ($t = 1$).

S.2 Platform investments in developers benefits

We consider the case in which the platforms engage in investments to improve the benefits of participating developers.

The payoff of a developer of type k interacting with consumers at platform $i \in 1, 2$ is

$$\pi_i^{Dev}(N_i^e, l_i, e_i) - k = \phi N_i^e + e_i - l_i - k,$$

where e_i is the participating benefits improved by platform i.

The profit of platform i is $\Pi_i = p_i N_i + l_i D_i - \gamma e_i^2$, where γ is an exogenous parameter that captures the difficulty of improving the benefits of participating developers.

At the beginning of the game, the platforms determine their levels of e_i. After that, the sequence of the game is the same as that in the benchmark model.

Uniform pricing The masses of consumers at platforms 1 and 2 are $N_i(D_1^e, D_2^e, p_1, p_2)$ in (3) ($i = 1, 2$), as the same as those in the main model. The mass of developers active on platform i is

$$D_i(N_i^e, l_i, e_i) = k_i(N_i^e, l_i, e_i) = \phi N_i^e + e_i - l_i.$$

Using the four equations, we obtain demands as a function of prices and fees:

$$\begin{aligned} N_i^*(p_i, p_{-i}, l_i, l_{-i}, e_i, e_{-i}) &= \frac{1}{2} + \frac{\theta(l_j - l_i + e_i - e_j) + p_j - p_i}{2(t - \theta\phi)}, \\ D_i^*(p_i, p_{-i}, l_i, l_{-i}, e_i, e_{-i}) &= \frac{\phi}{2} + \frac{(2t - \theta\phi)(e_i - l_i) - \theta\phi(e_j - l_j) + \phi(p_j - p_i)}{2(t - \theta\phi)}. \end{aligned}$$

The profit of platform i is $p_i N_i^*(\cdot) + l_i D_i^*(\cdot) - \gamma e_i^2$. Solving the first-order conditions leads to

$$\begin{aligned} p_i^*(e_i, e_{-i}) &= t - \frac{\phi(3\theta + \phi)}{4} \\ &\quad + \frac{(4t(\theta - 2\phi) + (\phi^2 + 4\theta\phi - \theta^2)\phi)e_i - (\theta + \phi)(4t - \phi(3\theta + \phi))e_j}{4(6t - (\theta^2 + 4\theta\phi + \phi^2))}, \\ l_i^*(e_i, e_{-i}) &= \frac{\phi - \theta}{4} + \frac{(12t - (\phi^2 + 8\theta\phi + 3\theta^2))e_i + (\theta^2 - \phi^2)e_j}{4(6t - (\theta^2 + 4\theta\phi + \phi^2))}. \end{aligned}$$

The profit of platform i is

$$\Pi_i^*(e_i, e_{-i}) = p_i^*(\cdot)N_i^*(p_i^*(\cdot), p_{-i}^*(\cdot), l_i^*(\cdot), l_{-i}^*(\cdot), e_i, e_{-i}) + l_i^*(\cdot)D_i^*(p_i^*(\cdot), p_{-i}^*(\cdot), l_i^*(\cdot), l_{-i}^*(\cdot), e_i, e_{-i}) - \gamma e_i^2.$$

At the investment stage, platform i determines e_i to maximize $\Pi_i^*(e_i, e_{-i})$. Solving the first-order conditions leads to

$$e_i = \frac{(\theta + \phi)(8t - (\theta^2 + 6\theta\phi + \phi^2))}{4(4\gamma - 1)(6t - (\theta^2 + 4\theta\phi + \phi^2))}.$$

The price is

$$p_i^U = t - \frac{\phi(3\theta + \phi)}{4} - \frac{\phi(\theta + \phi)(8t - (\theta^2 + 6\theta\phi + \phi^2))}{8(4\gamma - 1)(6t - (\theta^2 + 4\theta\phi + \phi^2))}.$$

The fee is

$$l_i^U = \frac{\phi - \theta}{4} + \frac{(\theta + \phi)(8t - (\theta^2 + 6\theta\phi + \phi^2))}{8(4\gamma - 1)(6t - (\theta^2 + 4\theta\phi + \phi^2))}.$$

The profits of the platforms are

$$\Pi_i^U = \frac{t}{2} - \frac{\theta^2 + 6\theta\phi + \phi^2}{16} - \frac{(\theta + \phi)^2(8t - (\theta^2 + 6\theta\phi + \phi^2))^2}{64(4\gamma - 1)(6t - (\theta^2 + 4\theta\phi + \phi^2))^2}.$$

The consumer surplus is

$$CS^U = w - \frac{5t - (\theta^2 + 4\theta\phi + \phi^2)}{4} + \frac{(\theta + \phi)^2(8t - (\theta^2 + 6\theta\phi + \phi^2))}{8(4\gamma - 1)(6t - (\theta^2 + 4\theta\phi + \phi^2))}.$$

The developer surplus is

$$PS^U = \frac{(\theta + \phi)^2(4(12\gamma - 1)t - (8\gamma - 1)(\theta + \phi)^2 - 16\theta\phi\gamma)^2}{64(4\gamma - 1)^2(6t - (\theta^2 + 4\theta\phi + \phi^2))^2}.$$

Personalized pricing The price schedules of platforms 1 and 2 are the same as in (9) and (10). The masses of consumers at platforms 1 and 2 are also the same as in the main model. We obtain the mass of developers active on platform i is

$$D_i^{**}(l_i, l_{-i}, e_i, e_{-i}) = \frac{\phi}{2} + \frac{(2t - \theta\phi)(e_i - l_i) - \theta\phi(e_j - l_j)}{2(t - \theta\phi)}.$$

Using this outcome, we obtain the price schedules of platforms 1 and 2:

$$
\begin{aligned}
p_1^{**}(l_1,l_2,e_1,e_2,x) &= \begin{cases} t(1-2x)+\dfrac{\theta(e_1-e_2+l_2-l_1)t}{t-\theta\phi} & \text{if } x\le \bar{x}^{**},\\ 0 & \text{if } x>\bar{x}^{**},\end{cases}\\
p_2^{**}(l_1,l_2,e_1,e_2,x) &= \begin{cases} t(2x-1)+\dfrac{\theta(e_2-e_1+l_1-l_2)t}{t-\theta\phi} & \text{if } x\ge \bar{x}^{**},\\ 0 & \text{if } x<\bar{x}^{**},\end{cases}\\
&\qquad\text{where } \bar{x}^{**}=N_1^{**}(l_1,l_2,e_1,e_2)=\frac{1}{2}+\frac{\theta(e_1-e_2+l_2-l_1)}{2(t-\theta\phi)}.
\end{aligned}
$$

$N_2^{**}(l_1,l_2,e_1,e_2)=1-N_1^{**}(l_1,l_2,e_1,e_2)$. The profits of platforms 1 and 2 are

$$
\begin{aligned}
&\int_0^{\bar{x}^{**}} p_1^{**}(l_1,l_2,e_1,e_2,x)dx+l_1D_1^{**}(l_1,l_2,e_1,e_2)-\gamma e_1^2,\\
&\int_{\bar{x}^{**}}^1 p_2^{**}(l_1,l_2,e_1,e_2,x)dx+l_2D_2^{**}(l_1,l_2,e_1,e_2)-\gamma e_2^2.
\end{aligned}
$$

Solving the first-order conditions leads to

$$
\begin{aligned}
l_i^P(e_1,e_2) &= \frac{(\phi-\theta)t-\theta\phi^2}{4t-3\theta\phi}+\frac{8t^3-2\theta(3\theta+8\phi)t^2+\theta^2\phi(5\theta+9\phi)t-\theta^3\phi^3}{(4t-3\theta\phi)(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2)}e_i\\
&\quad+\frac{\theta(2(\theta-\phi)t^2-\theta\phi(\theta-3\phi)t-\theta^2\phi^3)}{(4t-3\theta\phi)(4t^2-\theta(2\theta+5\phi)t+\theta^2\phi^2)}e_j.
\end{aligned}
$$

The profits of the platforms are

$$
\begin{aligned}
\Pi_1^*(e_1,e_2) &= \int_0^{\bar{x}^{**}} p_1^{**}(l_1^P(e_1,e_2),l_2^P(e_1,e_2),e_1,e_2,x)dx\\
&\quad+l_1^P(e_1,e_2)D_1^{**}(l_1^P(e_1,e_2),l_2^P(e_1,e_2),e_1,e_2)-\gamma e_1^2,\\
\Pi_2^*(e_1,e_2) &= \int_{\bar{x}^{**}}^1 p_2^{**}(l_1^P(e_1,e_2),l_2^P(e_1,e_2),e_1,e_2,x)dx\\
&\quad+l_2^P(e_1,e_2)D_2^{**}(l_1^P(e_1,e_2),l_2^P(e_1,e_2),e_1,e_2)-\gamma e_2^2.
\end{aligned}
$$

At the investment stage, platform i determines e_i to maximize $\Pi_i^*(e_i,e_{-i})$. Solving the first-order conditions leads to

$$
\begin{aligned}
e_i &= \frac{32(\theta+\phi)t^4-8\theta(\theta^2+11\theta\phi+10\theta^2)t^3}{2K_1}\\
&\quad+\frac{4\theta^2\phi(2\theta^2+18\theta\phi+17\phi^2)t^2-\theta^3\phi^3(17\theta+22\phi)t+2\theta^4\phi^5}{2K_1},
\end{aligned}
$$

where $K_1\equiv 32(4\gamma-1)t^4-16\theta(2(2\theta+11\phi)\gamma-(\theta+5\phi))t^3+2\theta^2\phi(4(12\theta+43\phi)\gamma-(11\theta+34\phi))t^2-\theta^3\phi^2(2(18\theta+69\phi)\gamma-(7\theta+22\phi))t+2\theta^4\phi^4(9\gamma-1)$. The fee is

$$
\begin{aligned}
l_i^P &= \frac{t(\phi-\theta)-\theta\phi^2}{4t-3\theta\phi}\\
&\quad+\frac{(t-\theta\phi)(8(\theta+\phi)t^2-8\theta\phi(\theta+\phi)t+\theta^2\phi^3)(4t^2-\theta(\theta+6\phi)t+2\theta^2\phi^2)}{(4t-3\theta\phi)K_1}.
\end{aligned}
$$

The profits of the platforms are

$$\Pi_i^P = \frac{t}{4} - \frac{((\theta-\phi)t+\theta\phi^2)(2(\theta+\phi)t-\theta\phi^2)}{2(4t-3\theta\phi)^2}, \\ -\frac{(8(\theta+\phi)t^2-8\theta\phi(\theta+\phi)t+\theta^2\phi^3)(4t^2-\theta(\theta+6\phi)t+2\theta^2\phi^2)^2K_2}{4(4t-3\theta\phi)^2K_1^2},$$

where $K_2 \equiv 32(4\gamma-1)(\theta-\phi)t^4 - 16\theta\phi(12(\theta-2\phi)\gamma-(3\theta-5\phi))t^3 + 4\theta^2\phi^2(6(3\theta-17\phi)\gamma-(5\theta-17\phi))t^2 + 2\theta^3\phi^3(90\phi\gamma+(\theta-11\phi))t - \theta^4\phi^5(27\gamma-2)$. The consumer surplus is

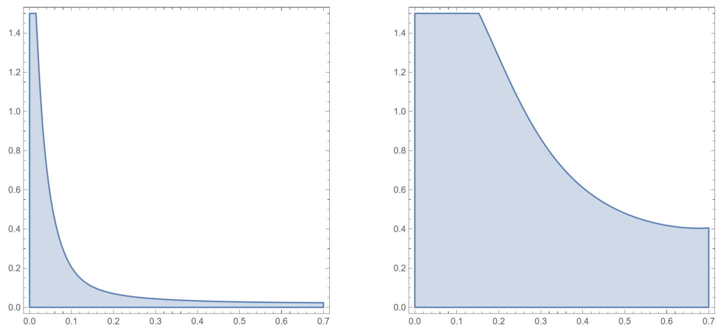
$$CS^P = w - \frac{12t^2-\theta(4\theta+13\phi)t+2\theta^2\phi^2}{4(4t-3\theta\phi)} \\ -\frac{\theta(2t-\theta\phi)(8(\theta+\phi)t^2-8\theta\phi(\theta+\phi)t+\theta^2\phi^3)(4t^2-\theta(\theta+6\phi)t+2\theta^2\phi^2)}{2(4t-3\theta\phi)K_1}.$$

The developer surplus is:

$$PS^P = \left(\frac{32(\theta+\phi)\gamma t^4 - 2\theta(8(\theta^2+5\theta\phi+5\phi^2)\gamma-\theta(\theta-\phi))t^3}{K_1} \right. \\ \left. + \frac{\theta^2\phi(2(2\theta+5\phi)(3\theta+7\phi)\gamma+\theta(3\phi-\theta))t^2-\theta^3\phi^3((12\theta+25\phi)\gamma+\theta)t+3\theta^4\phi^5\gamma}{K_1}\right)^2.$$

The average consumer price in personalized pricing is $t/2$ because of the symmetry of the platforms. Calculating $t/2 - p_i^U$ and $l_i^P - l_i^U$ leads to the following proposition:

Proposition B1. *The average consumer price in personalized pricing is higher than the price in uniform pricing when ϕ is larger than a threshold value. The fees in personalized pricing are higher than those in uniform pricing when ϕ is larger than a threshold value.*



($\gamma = 10$) ($\gamma = 1$)

Figure 2: Comparison of fees ($t = 1$).
$l_i^P > l_i^U$ in the colored area. Horizontal: θ, Vertical: ϕ.

We also obtain the following proposition, which is similar to Proposition 2:

Proposition B2. *Personalized pricing improves the surplus of developers and the total surplus.*

Related to the effect of personalized pricing on profits and consumer surplus, we show two numerical results in which (i) $\gamma = 10$ and (ii) $\gamma = 1$.

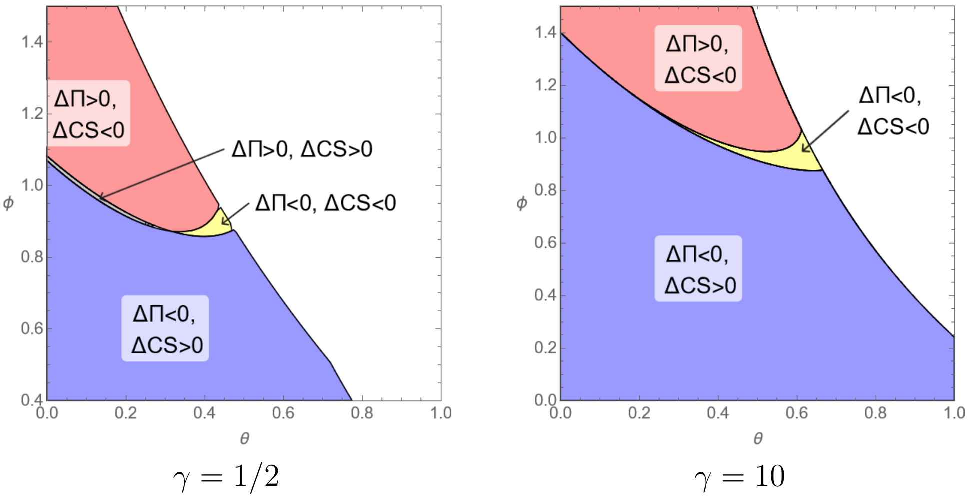


Figure 3: Comparison of profits and consumer surplus ($t = 1$).
Horizontal: θ, Vertical: ϕ.

S.3 Single-Homing Developers

In this extension, we allow for developers to single-home while the consumer side is as in the benchmark model.

Developers, in this setting, are uniformly distributed on a Hotelling line according to their relative preference y for platform 2 over platform 1. A developer of type y incurs a mismatch cost of ty and $t(1-y)$ when interacting with consumers, respectively, at platforms 1 and 2, where t is the transport cost. The surplus of a developer of type y when purchasing platform 1's product or platform 2's product is given as:

$$\pi_1(l_1, N_1^e, y) = \quad v + \phi N_1^e - l_1 - ty, \quad \text{on platform 1,} \tag{B.4}$$
$$\pi_2(l_2, N_2^e, y) = \quad v + \phi N_2^e - l_2 - t(1-y), \quad \text{on platform 2,} \tag{B.5}$$

where $v(>0)$ is the common intrinsic utility of the developer, ϕN_i^e represents the expected value developers derive from interacting with consumers, l_i is the developer fee charged by platform i.
The profit of each platform i and the timing remains as in the benchmark model. We impose the following technical restrictions.

Assumption 2. *The exogenous parameters, $t > 0$, $\theta > 0$, and $\phi > 0$, satisfy $t > \underline{t} \equiv \max\{(\theta+\phi)/2, \sqrt{\theta(\theta+2\phi)/2}\}$. Further, we assume that the intrinsic values of consumers and developers are sufficient high such that the market is covered.*

These restrictions ensure that the second-order conditions are satisfied in both pricing regimes and that the markets are covered.

Uniform pricing

The location of indifferent consumers is denoted by $\bar{x}$ as in equation (3).

As developers single-home, they must decide which platform to affiliate with. The location of each indifferent developer is denoted by $\bar{y} = \frac{1}{2} + \frac{\phi(N_1^e - N_2^e) - (l_1 - l_2)}{2t}$. Thus, the mass of developers on platform 1 and f_2 are given as

$$D_1(N_1^e, N_2^e, l_1, l_2) = \bar{y}, \;\; D_2(N_2^e, N_1^e, l_2, l_1) = 1 - \bar{y}. \tag{B.6}$$

Imposing fulfilled expectations equilibrium, the expected mass of developers and consumers must match the realized demands. By imposing $N_i^e = N_i^\star$ and $D_i^e = D_i^\star$ in equations (3) and (B.6) and solving for the mass of consumers and the mass of developers at the two platforms, we obtain demands as a function of fees, as presented below for $(i = 1, 2)$:

$$\begin{aligned} N_i^\star(p_i, p_{-i}, l_i, l_{-i}) &= \frac{1}{2} - \frac{(p_i - p_{-i}) + \theta(l_i - l_{-i})}{2(t^2 - \theta\phi)}, \\ D_i^\star(l_i, l_{-i}, p_i, p_{-i}) &= \frac{1}{2} - \frac{(l_i - l_{-i}) + \phi(p_i - p_{-i})}{2(t^2 - \theta\phi)}. \end{aligned} \tag{B.7}$$

In stage 1, each platform $i \in \{1, 2\}$ sets prices and fees to maximize its profits, given as

$$\max_{l_i, p_i} \Pi_i(p_i, p_{-i}, l_i, l_{-i}) = p_i N_i^\star(\cdot) + l_i D_i^\star(\cdot).$$

Differentiating the profit of each platform $i \in 1, 2$ with respect to p_i and l_i and solving yields the equilibrium fees as presented below.

$$p_i^U = t - \phi, \quad l_i^U = t - \theta. \tag{B.8}$$

Substituting the equilibrium fees presented in equation (8) into the calculations for profits, demands, consumer surplus, and producer surplus yields the following outcome. The equilibrium profits, the equilibrium mass of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:

$$\begin{aligned} \pi_i^U \quad &= t - \frac{(\theta + \phi)}{2}, \quad N_i^U = \frac{1}{2}, \quad D_i^U = \frac{1}{2}, \\ CS^U \quad &= \int_0^{N_1^U} (w + \theta D_1^U - tx - p_1^U) dx + \int_{N_1^U}^1 (w + \theta D_2^U - t(1 - x) - p_2^U) dx \\ &= w + \phi + \frac{\theta}{2} - \frac{5t}{4}, \\ PS^U \quad &= \int_0^{D_1^U} (v + \phi N_1^U - l_1^U - ty) dy + \int_{D_1^U}^1 (v + \phi N_2^U - l_2^U - t(1 - y)) dy \\ &= v + \theta + \frac{\phi}{2} - \frac{5t}{4}. \end{aligned} \tag{B.9}$$

Personalized pricing

Under personalized pricing, the expression for consumer price and their demands are as in equations (9) and (10).

Considering the fees charged to developers, the expected number of developers must align with the actual number, denoted as $D_i^e = D_i^{\star\star}$ $(i = 1, 2)$, under the conditions $N_i^e = N_i^{\star\star}$. Utilizing these conditions, comprising four equations and solving, determines that the mass of developers active in each platform i is solely influenced by the developer participation fees.

$$D_i^{\star\star}(l_i, l_{-i}) = \frac{1}{2} \left(1 - \frac{t(l_i - l_{-i})}{t^2 - \theta\phi} \right) \text{ for } i = 1, 2. \tag{B.10}$$

By substituting the mass of developers as presented in equation (B.10), we derive the actual price schedules and the mass of consumers for platform i as a function of developer fees:

$$p_1^{\star\star}(l_1,l_2,x)=\begin{cases}\dfrac{\theta(l_2-l_1)t}{t^2-\theta\phi}+t(1-2x) & \text{if } x\le\bar{x}^{\star\star},\\ 0 & \text{if } x>\bar{x}^{\star\star},\end{cases}\tag{B.11}$$

$$p_2^{\star\star}(l_2,l_1,x)=\begin{cases}0 & \text{if } x\le\bar{x}^{\star\star},\\ \dfrac{\theta(l_2-l_1)t}{t^2-\theta\phi}+t(2x-1), & \text{if } x>\bar{x}^{\star\star},\end{cases}\tag{B.12}$$

$$N_1^{\star\star}(l_1,l_2)=\frac{1}{2}\left(1+\frac{\theta(l_2-l_1)}{t^2-\theta\phi}\right),\quad N_2^{\star\star}(l_2,l_1)=\frac{1}{2}\left(1+\frac{\theta(l_1-l_2)}{t^2-\theta\phi}\right),\tag{B.13}$$

where $\bar{x}^{\star\star}(l_1,l_2)=\bar{x}_1(D_1^{\star\star}(\cdot),D_2^{\star\star}(\cdot))$.

The profit expression of platforms are as in the benchmark. Differentiating the profit of each platform i with respect to the fee l_i and solving simultaneously yields the equilibrium fees as follows.

$$l_1^P=l_2^P=t-\theta-\frac{\theta\phi}{t}.\tag{B.14}$$

Substituting these equilibrium fees into the personalized pricing schedules yields prices as in the benchmark in equation (17). Substituting the equilibrium fees as in equations (B.14) and (17) yields the following outcome:

The equilibrium profits, the equilibrium numbers of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:

$$\begin{aligned}\Pi_i^P &= \frac{1}{4}\left(3t-2\theta-\frac{2\theta\phi}{t}\right),\quad N_i^P=\frac{1}{2},\quad D_i^P=\frac{1}{2},\\ CS^P &= \int_0^{\bar{x}^P}(w+\theta D_1^P-tx-p_1^P(x))dx+\int_{\bar{x}^P}^1(w+\theta D_2^P-t(1-x)-p_2^P(x))dx\\ &= w+\frac{\theta}{2}-\frac{3t}{4},\\ PS^P &= \int_0^{D_1^P} v+\phi N_1^P-l_1^P-ty)dy+\int_{D_1^P}^1(v+\phi N_2^P-l_2^P-t(1-y))dy\\ &= v+\theta+\frac{\phi}{2}+\frac{\theta\phi}{t}-\frac{5t}{4}.\end{aligned}$$

Welfare effects of pricing regimes

We define the total surplus as the sum of platform profits, developer surplus, and consumer surplus.

$$TS^k=\sum_{i=1}^{2}\Pi_i^k+CS^k+PS^k \text{ for } k\in\{U,\ P\}.$$

We obtain the following outcomes, which are summarized as in Proposition 2:

$$\Delta PS=PS^P-PS^U \;=\; \frac{\theta\phi}{t}>0,\tag{B.15}$$

$$\Delta TS=TS^P-TS^U \;=\; 0.\tag{B.16}$$

Next, we examine the effects of personalized pricing on profits and consumer surplus. To do so, we define the difference in profits and consumer surplus between the two regimes as $\Delta\Pi=\Pi_i^P-\Pi_i^U=\frac{1}{4}\left(-\frac{2\theta\phi}{t}-t+2\phi\right)$ and $\Delta CS=CS^P-CS^U=\frac{t-2\phi}{2}$. Figure 4 illustrates the regions in which personalized pricing enhances profits or consumer surplus.

The following relationship holds.

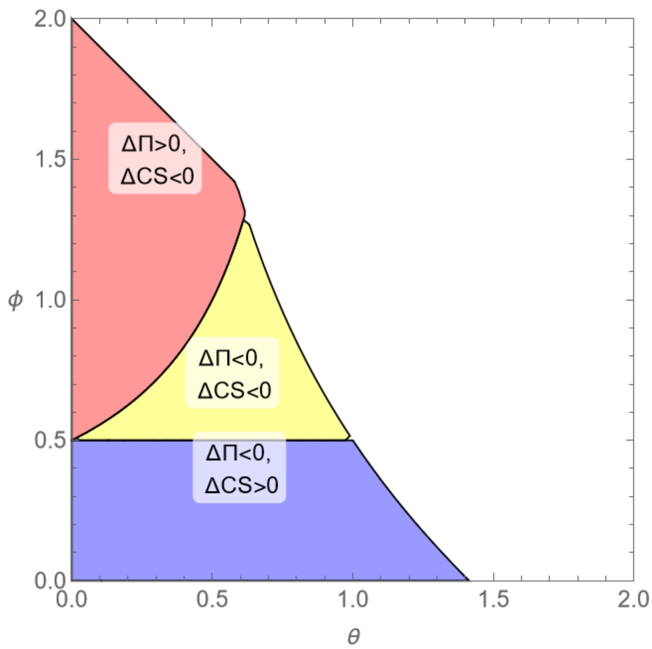


Figure 4: Comparison of profits and consumer surplus ($t = 1$).

- When $0 < \phi < \phi_1 = \frac{t}{2}$, consumer surplus (platform profit) under personalized pricing is higher (lower) higher than under uniform pricing — i.e., $\Delta CS > 0$ and $\Delta\Pi < 0$.
- When $\phi_1 < \phi < \phi_2 = \frac{t^2}{2(t-\theta)}$, consumer surplus and platform profits under personalized pricing are lower than under uniform pricing — i.e., $\Delta CS < 0$ and $\Delta\Pi < 0$.
- When $\phi_2 < \phi$, consumer surplus (platform profit) under personalized pricing is lower (higher) than under uniform pricing — i.e., $\Delta CS < 0$ and $\Delta\Pi > 0$.

S.4 Multi-Homing Developers

In this extension, we allow developers to multi-home on the two platforms.

Towards this, we assume developers are distributed according to their investment cost of developing content k that is not specific to a platform. Therefore, after investment k, developers will visit both platforms if

$$\pi_{MH}(l_1, l_2, N_1^e, N_2^e) - k = \phi(N_1^e + N_2^e) - l_1 - l_2 - k > 0.$$

In this setting, we conjecture that all developers multi-home and we will confirm this later.

The profit of each platform i and the timing remains as in the benchmark model.

We impose the following technical restrictions.

Assumption 3. *The exogenous parameters, $t > 0$, $0 < \theta \leq 1$ and $0 < \phi < 3$. The intrinsic value of consumers is sufficiently high such that the market is covered.*

These restrictions ensure that the second-order conditions are satisfied in both pricing regimes and markets are covered.

Uniform pricing

As we conjecture that all developers multi-home, they must decide whether to invest and join both platforms. Therefore, $\pi_{MH}(\cdot) \geq k$ which yields the mass of multi-homing developers is

$$D_{MH}(N_1^e, N_2^e, l_1, l_2) = \phi(N_1^e + N_2^e) - l_1 - l_2. \tag{B.17}$$

The location of indifferent consumers is denoted by $\bar{x}$ as in equation (3). There is a minor difference from the benchmark. As all developers multi-home, consumers expect to find the same developers on both platforms and therefore, the expectations of the mass of developers are just equal to the mass of multi-homers — i.e., $D_i^e = D_{MH}^e$.

Imposing fulfilled expectations equilibrium, the expected mass of developers and consumers must match the realized demands. By imposing $N_i^e = N_i^\star$ and $D_i^e = D_{MH}^\star$ in equations (3) and (B.17) and solving for the mass of consumers and the mass of developers at the two platforms, we obtain demands as a function of fees, as presented below for $(i = 1, 2)$:

$$N_i^\star(p_i, p_{-i}) = \frac{1}{2} - \frac{(p_i - p_{-i})}{2t}, \; D_{MH}^\star(l_i, l_{-i}) = \phi - l_1 - l_2. \tag{B.18}$$

Under this conjecture, note that demands depend only on their own prices and not on the price charged to the other side. This absence of consumer (developer) fees affecting developer (consumer) demand suggests that consumers and developers under this conjecture are not impacted by network effects, and the market transforms into a classical one-sided market.

In stage 1, each platform $i \in \{1, 2\}$ sets prices and fees to maximize its profits, given as

$$\max_{l_i, p_i} \Pi_i(p_i, p_{-i}, l_i, l_{-i}) = p_i N_i^\star(\cdot) + l_i D_i^\star(\cdot).$$

Differentiating the profit of each platform $i \in 1, 2$ with respect to p_i and l_i and solving yields the equilibrium fees as presented below.

$$p_i^U = t, \;\; l_i^U = \frac{\phi}{3}. \tag{B.19}$$

As expected, notice that the fee charged to consumers is exactly equal to the outcome of a classical Hotelling setup. While the fee charged to developers is equal to the fee charged by complementary good suppliers (in our setting platforms). Specifically, the total fee to developers is $l_1^U + l_2^U = \frac{2\phi}{3}$ is greater than the fee charged if the two platforms could coordinate and avoid the Cournot externality. Specifically, if the two platforms could coordinate their fees to developers, they would set total fees equal to $\phi/2 < \frac{2\phi}{3}$. Essentially, the pricing on the two sides is as in traditional one-sided markets.

Substituting the equilibrium fees presented in equation (8) into the calculations for profits, demands, consumer surplus, and producer surplus yields the following outcome. The equilibrium profits, the equilibrium mass of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:

$$\begin{aligned}
\pi_i^U &= \frac{t}{2} + \frac{\phi^2}{9}, \;\; N_i^U = \frac{1}{2}, \;\; D_{MH}^U = \frac{\phi}{3}, \\
CS^U &= \int_0^{N_1^U} (w + \theta D_{MH}^U - tx - p_1^U) dx + \int_{N_1^U}^1 (w + \theta D_{MH}^U - t(1-x) - p_2^U) dx \\
&= w + \frac{\theta\phi}{3} - \frac{5t}{4}, \\
PS^U &= \int_0^{D_{MH}^U} (\phi(N_1^U + N_2^U) - l_1^U - l_2^U - k) dk = \frac{\phi^2}{18}.
\end{aligned} \tag{B.20}$$

Personalized pricing

Under personalized pricing, the expression for consumer price and their demands are as in equations (9) and (10). As all developers multi-home — i.e., $D_i^e = D_{MH}$, we appropriately adjust the prices and the demand expressions.

Considering the fees charged to developers, the expected number of developers must align with the actual number, denoted as $D_i^e = D_{MH}^{\star\star}$ $(i = 1, 2)$, under the conditions $N_i^e = N_i^{\star\star}$. Utilizing these conditions, comprising four equations and solving, determines that the mass of developers active in each platform i is solely influenced by the developer participation fees.

$$D_{MH}^{\star\star}(l_i, l_{-i}) = \phi - l_1 - l_2. \tag{B.21}$$

Note that the expression for the mass of multi-homers is as in the uniform pricing case. By substituting the mass of developers as presented in equation (B.21), we derive the actual price schedules and the mass of consumers for platform i as a function of developer fees:

$$p_1^{\star\star}(l_1, l_2, x) = \begin{cases} t(1-2x) & \text{if } x \leq \bar{x}^{\star\star}, \\ 0 & \text{if } x > \bar{x}^{\star\star}, \end{cases} \tag{B.22}$$

$$p_2^{\star\star}(l_2, l_1, x) = \begin{cases} 0 & \text{if } x \leq \bar{x}^{\star\star}, \\ t(2x-1), & \text{if } x > \bar{x}^{\star\star}, \end{cases} \tag{B.23}$$

$$N_1^{\star\star}(l_1, l_2) = \frac{1}{2}, \quad N_2^{\star\star}(l_2, l_1) = \frac{1}{2}, \tag{B.24}$$

where $\bar{x}^{\star\star}(l_1, l_2) = \bar{x}_1(D_{MH}^{\star\star}(\cdot))$.

The profit expression of platforms is as in the benchmark. Differentiating the profit of each platform i with respect to the fee l_i and solving simultaneously yields the equilibrium fees as follows.

$$l_1^P = l_2^P = \frac{\phi}{3}. \tag{B.25}$$

Substituting these equilibrium fees into the personalized pricing schedules yields prices as in the benchmark in equation (17). Substituting the equilibrium fees as in equations (B.25) and (17) yields the following outcome:
The equilibrium profits, the equilibrium numbers of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:

$$\begin{aligned}
\Pi_i^P &= \frac{t}{4} + \frac{\phi^2}{9}, \quad N_i^P = \frac{1}{2}, \quad D_{MH}^P = \frac{\phi}{3}, \\
CS^P &= \int_0^{\bar{x}^P} (w + \theta D_{MH}^P - tx - p_1^P(x))dx + \int_{\bar{x}^P}^1 (w + \theta D_{MH}^P - t(1-x) - p_2^P(x))dx \\
&= v + \frac{\phi\theta}{3} - \frac{3t}{4}, \\
PS^P &= \int_0^{D_{MH}^P} (\phi(N_1^P + N_2^P) - l_1^P - l_2^P - k)dk = \frac{\phi^2}{18}.
\end{aligned}$$

Welfare effects of pricing regimes

We define the total surplus as the sum of platform profits, developer surplus, and consumer surplus.

$$TS^k = \sum_{i=1}^{2} \Pi_i^k + CS^k + PS^k \text{ for } k \in \{U,\ P\}.$$

Under Assumptions 3, we obtain the following outcomes:

$$\Delta PS = PS^P - PS^U = 0, \tag{B.26}$$

$$\Delta CS = CS^P - CS^U = \frac{t}{2}, \tag{B.27}$$

$$\Delta \Pi_i = \Pi_i^P - \Pi_i^U = -\frac{t}{4} \tag{B.28}$$

$$\Delta TS = TS^P - TS^U = 0. \tag{B.29}$$

S.5 Partial multi-homing developers

In this section, we consider the case that only part of the developers multi-home.

Towards this, we follow the same setting in Section S.3 with the following modification. The surplus of a developer of type y when purchasing platform 1's product, platform 2's product, or both products is given as:

$$\begin{aligned} \pi_1(l_1, N_1^e, y) &= v + \phi N_1^e - l_1 - ty && \text{on platform 1,} \\ \pi_2(l_2, N_2^e, y) &= v + \phi N_2^e - l_2 - t(1-y) && \text{on platform 2,} \\ \pi_m(l_1, l_2, N_1^e, N_2^e, y) &= v + \delta + \phi(N_1^e + N_2^e) - l_1 - l_2 - t && \text{under multi-homing,} \end{aligned} \tag{B.30}$$

where δ denotes the extra benefit the developer gains from purchasing the second product. The profit of each platform and the timing remain as in the benchmark model. We make the following assumption to ensure partial multi-homing among developers.

Assumption 4. $\frac{1}{4}(2\delta+\theta+\phi) \leq t < \frac{1}{2}(2\delta+\theta+\phi)$, *so that only part of the developers multi-home.*

Uniform pricing

The location of indifferent consumers is denoted by $\bar{x}$ as in equation (3).

The developers choose the optimal action among participating in platform 1, platform 2, and both platforms. Since we assume that part of the developers multi-home, there are two kinds of indifferent developers. The location of developers who are indifferent between purchasing from platform 1 and multi-homing is denoted by $\bar{y}_1 = 1 - \frac{\phi N_2^e + \delta - l_2}{t}$. The location of developers who are indifferent between purchasing from platform 2 and multi-homing is denoted by $\bar{y}_2 = \frac{\phi N_1^e + \delta - l_1}{t}$. Thus, the mass of developers on platform 1 and 2 are given as

$$D_1(N_1^e, N_2^e, l_1, l_2) = \bar{y}_2, \;\; D_2(N_2^e, N_1^e, l_2, l_1) = 1 - \bar{y}_1. \tag{B.31}$$

Imposing fulfilled expectations equilibrium, the expected mass of developers and consumers must match the realized demands. By imposing $N_i^e = N_i^\star$ and $D_i^e = D_i^\star$ in equations (3) and (B.31) and solving for the mass of consumers and the mass of developers at the two platforms, we obtain demands as a function of fees, as presented below for $(i = 1, 2)$:

$$\begin{aligned} N_i^\star(p_i, p_{-i}, l_i, l_{-i}) &= \frac{1}{2} - \frac{t(p_i - p_{-i}) + \theta(l_i - l_{-i})}{2(t^2 - \theta\phi)}, \\ D_i^\star(l_i, l_{-i}, p_i, p_{-i}) &= \frac{2\delta + \phi}{2t} - \frac{\phi t(p_i - p_{-i}) + (2t^2 - \theta\phi)l_i - \theta\phi l_{-i}}{2t(t^2 - \theta\phi)}. \end{aligned} \tag{B.32}$$

In stage 1, each platform $i \in \{1, 2\}$ sets prices and fees to maximize its profits, given as

$$\max_{l_i, p_i} \Pi_i(p_i, p_{-i}, l_i, l_{-i}) = p_i N_i^\star(\cdot) + l_i D_i^\star(\cdot).$$

Differentiating the profit of each platform $i \in 1, 2$ with respect to p_i and l_i and solving yields the equilibrium fees as presented below.

$$p_i^U = t - \frac{\phi(2\delta + 3\theta + \phi)}{4t}, \quad l_i^U = \frac{2\delta - \theta + \phi}{4}. \tag{B.33}$$

Substituting the equilibrium fees presented in equation (B.33) into the calculations for profits, demands, consumer surplus, and producer surplus yields the following outcome. The equilibrium profits, the equilibrium mass of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are:

$$\begin{aligned}
\pi_i^U &= \frac{t}{2} + \frac{\delta^2}{4t} - \frac{\theta^2 + 6\theta\phi + \phi^2}{16t}, \quad N_i^U = \frac{1}{2}, \quad D_i^U = \frac{2\delta + \theta + \phi}{4t}, \\
CS^U &= \int_0^{N_1^U} (w + \theta D_1^U - tx - p_1^U)dx + \int_{N_1^U}^1 (w + \theta D_2^U - t(1-x) - p_2^U)dx \\
&= v - \frac{5}{4}t + \frac{\theta^2 + 4\theta\phi + \phi^2 + 2\delta(\theta + \phi)}{4t}, \\
PS^U &= \int_0^{1-D_2^U} (v + \phi N_1^U - l_1^U - ty)dy + \int_{1-D_2^U}^{D_1^U} (v + \delta + \phi(N_1^U + N_2^U) - l_1^U - l_2^U - t)dy \\
&+ \int_{D_1^U}^1 (v + \phi N_2^U - l_2^U - t(1-y))dy = v - \delta + \frac{(2\delta + \theta + \phi)^2}{16t}.
\end{aligned} \tag{B.34}$$

Personalized pricing

Under personalized pricing, the expression for consumer price and their demands are as in equations (9) and (10).

Considering the fees charged to developers, the expected number of developers must align with the actual number, denoted as $D_i^e = D_i^{\star\star}$ $(i = 1, 2)$, under the conditions $N_i^e = N_i^{\star\star}$. Utilizing these conditions, comprising four equations and solving, determines that the mass of developers active in each platform i is solely influenced by the developer participation fees.

$$D_i^{\star\star}(l_i, l_{-i}) = \frac{(t^2 - \theta\phi)(2\delta + \phi) - (2t^2 - \theta\phi)l_i + \theta\phi l_{-i}}{2t(t^2 - \theta\phi)} \quad \text{for } i = 1, 2. \tag{B.35}$$

By substituting the mass of developers as presented in equation (B.35), we derive the actual price schedules and the mass of consumers for platform i as a function of developer fees:

$$p_1^{\star\star}(l_1, l_2, x) = \begin{cases} \dfrac{\theta(l_2 - l_1)t}{t^2 - \theta\phi} + t(1 - 2x) & \text{if } x \leq \bar{x}^{\star\star}, \\ 0 & \text{if } x > \bar{x}^{\star\star}, \end{cases} \tag{B.36}$$

$$p_2^{\star\star}(l_2, l_1, x) = \begin{cases} 0 & \text{if } x \leq \bar{x}^{\star\star}, \\ \dfrac{\theta(l_1 - l_2)t}{t^2 - \theta\phi} + t(2x - 1), & \text{if } x > \bar{x}^{\star\star}, \end{cases} \tag{B.37}$$

$$N_1^{\star\star}(l_1, l_2) = \frac{1}{2}\left(1 + \frac{\theta(l_2 - l_1)}{t^2 - \theta\phi}\right), \quad N_2^{\star\star}(l_2, l_1) = \frac{1}{2}\left(1 + \frac{\theta(l_1 - l_2)}{t^2 - \theta\phi}\right), \tag{B.38}$$

where $\bar{x}^{\star\star}(l_1, l_2) = \bar{x}_1(D_1^{\star\star}(\cdot), D_2^{\star\star}(\cdot))$. This outcome is the same as those in Section S.3.

The profit expression of platforms is as in the benchmark. Differentiating the profit of each platform i with respect to the fee l_i and solving simultaneously yields the equilibrium fees as follows.

$$l_1^P = l_2^P = \frac{t^2(2\delta - \theta + \phi) - \theta\phi(2\delta + \phi)}{4t^2 - 3\theta\phi}. \tag{B.39}$$

Substituting these equilibrium fees into the personalized pricing schedules yields prices as in the benchmark in equation (17). Substituting the equilibrium fees as in equations (B.39) and (17) yields the following outcome: The equilibrium profits, the equilibrium numbers of consumers and developers, the equilibrium consumer surplus, and the equilibrium surplus of developers are

$$\begin{aligned}
\Pi_i^P &= \frac{t}{4} + \frac{(2\delta+\phi)^2(t^2-\theta\phi)(2t^2-\theta\phi)}{2t(4t^2-3\theta\phi)^2} - \frac{t^2\theta^2(2t^2+\phi(2\delta+\phi))}{2t(4t^2-3\theta\phi)^2}, \quad N_i^P = \frac{1}{2},\\
D_i^P &= \frac{2t^2(2\delta+\phi+\theta)-\phi\theta(2\delta+\phi)}{t(8t^2-6\theta\phi)},\\
CS^P &= \int_0^{\bar{x}^P}(w+\theta D_1^P - tx - p_1^P(x))dx + \int_{\bar{x}^P}^1 (w+\theta D_2^P - t(1-x) - p_2^P(x))dx\\
&= w - \frac{3}{4}t + \frac{\theta(2t^2(2\delta+\theta+\phi)-\theta\phi(2\delta+\phi))}{2t(4t^2-3\theta\phi)},\\
PS^P &= \int_0^{1-D_2^P}(v+\phi N_1^P - l_1^P - ty)dy\\
&\quad + \int_{1-D_2^P}^{D_1^P}(v+\delta+\phi(N_1^P+N_2^P)-l_1^P-l_2^P-t)dy + \int_{D_1^P}^1 (v+\phi N_2^P - l_2^P - t(1-y))dy\\
&= v + \frac{1}{4t(4t^2-3\theta\phi)^2} \times \Big\{\theta^2\phi^2(2\delta+\phi)^2 - 36t\delta\theta^2\phi^2 - 4t^2\theta\phi(2\delta+\phi)(2\delta+\theta+\phi)\\
&\qquad +96t^3\delta\theta\phi + 4t^4(2\delta+\theta+\phi)^2 - 64t^5\delta\Big\}.
\end{aligned}$$

Welfare effects of pricing regimes

Comparing consumer surplus in the two regimes, we have

$$\Delta PS = PS^P - PS^U = \frac{\theta\phi(2\delta+3\theta+\phi)\left(8t^2(2\delta+\theta+\phi)-\theta\phi(10\delta+3\theta+5\phi)\right)}{16t\left(4t^2-3\theta\phi\right)^2} > 0,$$

$$\Delta CS = CS^P - CS^U = \frac{\theta\phi(\theta+3\phi)(2\delta+3\theta+\phi)-2t^2\phi(4\delta+9\theta+2\phi)+8t^4}{4\left(4t^3-3\theta t\phi\right)},$$

$$\Delta\Pi_i = \Pi_i^P - \Pi_i^U = \frac{\begin{pmatrix} 32t^4\phi(2\delta+6\theta+\phi) - 4\theta t^2\phi\left(4\delta(\theta+6\phi)+6\theta^2+47\theta\phi+12\phi^2\right) \\ -\theta^2\phi^2(4\delta^2-9\theta^2-32\delta\phi-54\theta\phi-17\phi^2)-64t^6\end{pmatrix}}{16t\left(4t^2-3\theta\phi\right)^2}.$$

The outcome leads to Figure 5.

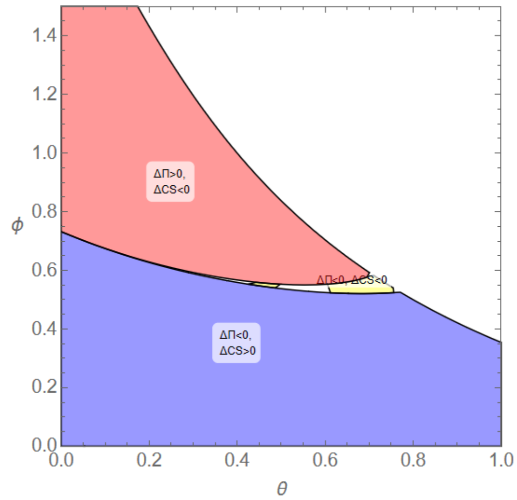


Figure 5: Comparison of profits and consumer surplus ($t = 1$ and $\delta = 1$).

S.6 A foundation of exogenous ϕ and θ in Section S.1

We show a micro-foundation in which developers set their prices for consumers at a level that is independent of commission fees r_i. We use a technique in Anderson and Bedre-Defolie (2025, mimeographed), who provide a useful micro-foundation for the independence between developers' prices and percentage-based commission fees.

We explain the detail of the micro-foundation. Developers are symmetric except for their outside options k, which is the same as that in our paper. The distribution of k follows the assumption in our paper. Each developer's app is a monopolistic product in its market, and consumers are willing to consume in all app markets. Anderson and Bedre-Defolie (2025) mention that this assumption could be interpreted as *"time spent on an app or in-app purchases (micro-payments)."* Consumers pay a price ρ for unit consumption to the app developer and also incur an exogenous cost $\kappa(> 0)$, which reflects an intrinsic cost of a transaction. A consumer gains from purchasing q units of app at price ρ by

$$u_A(q, \rho) = \begin{cases} -\mu + 2\sqrt{q} - (\kappa + \rho)q & \text{if } q > 0, \\ 0 & \text{if } q = 0, \end{cases}$$

where μ is a fixed cost to install an app.

Under the transactions between app developers and consumers on the platforms, we consider the following timing. Platforms simultaneously offer prices p_i and commission fees r_i to consumers and developers, respectively. When platforms employ personalized prices, which are private, we replace p_i with $p_i(x)$. Consumers and developers, respectively, form expectations for the masses of developers and consumers in each platform and then decide to affiliate with platforms. Observing the realized masses, developers set app prices. After that, each consumer chooses the amount of app consumption.

In a given app market with in-app purchase price ρ, a consumer chooses her optimal consumption level by solving the following problem:

$$\max_q \; u(q, \rho) = \max_q \begin{cases} -\mu + 2\sqrt{q} - (\kappa + \rho)q & \text{if } q > 0, \\ 0 & \text{if } q = 0, \end{cases}$$

The maximization problem leads to

$$q^*(\rho) = \begin{cases} \dfrac{1}{(\kappa+\rho)^2} & \text{if } \rho \leq \dfrac{1}{\mu} - \kappa, \\ 0 & \text{if } \rho > \dfrac{1}{\mu} - \kappa. \end{cases}$$

The indirect utility from transacting with an app is

$$v(\rho) = \begin{cases} \dfrac{1}{(\kappa+\rho)} - \mu & \text{if } \rho \leq \dfrac{1}{\mu} - \kappa, \\ 0 & \text{if } \rho > \dfrac{1}{\mu} - \kappa. \end{cases}$$

Given that the consumers belong to the platform i that charges r_i, a developer sets ρ. The total demand for the developer is the number of participating consumers N_i times each consumer's demand for the app, $q^*(\rho)$, because all consumers have the same utility function $u(q, \rho)$. Each monopolistic developer in an app market chooses ρ to maximize the following profit:

$$\pi(\rho, r_i) = (1-r_i)\rho N_i q^*(\rho) = \begin{cases} (1-r_i)N_i \dfrac{\rho}{(\kappa+\rho)^2} & \text{if } \rho \leq \dfrac{1}{\mu} - \kappa, \\ 0 & \text{if } \rho > \dfrac{1}{\mu} - \kappa. \end{cases}$$

The first-order condition of each developer is

$$\frac{\partial \pi(\rho, r_i)}{\partial \rho} = \begin{cases} (1-r_i)N_i \dfrac{(\kappa+\rho)^2 - 2\rho(\kappa+\rho)}{(\kappa+\rho)^4} & \text{if } \rho \leq \dfrac{1}{\mu} - \kappa, \\ 0 & \text{if } \rho > \dfrac{1}{\mu} - \kappa. \end{cases}$$

The first-order condition leads to

$$\rho^* = \begin{cases} \kappa & \text{if } \kappa \leq \dfrac{1}{2\mu}, \\ \dfrac{1}{\mu} - \kappa & \text{if } \kappa > \dfrac{1}{2\mu}, \end{cases}$$

The equilibrium app price is independent of r_i. The per-transaction profit of each developer is

$$\phi \equiv \rho^* q(\rho^*) = \begin{cases} \dfrac{1}{4\kappa} & \text{if } \kappa \leq \dfrac{1}{2\mu}, \\ \mu(1-\mu\kappa) & \text{if } \kappa > \dfrac{1}{2\mu}, \end{cases}$$

The equilibrium indirect utility from one transaction with a developer is

$$\theta \equiv v(\rho^*) = \begin{cases} \dfrac{1}{2\kappa} - \mu & \text{if } \kappa \leq \dfrac{1}{2\mu}, \\ 0 & \text{if } \kappa > \dfrac{1}{2\mu}. \end{cases}$$

The values of ϕ and θ in this section are related to those in Section S.1.

Anticipating the indirect utility and the per-transaction profit on the platform, consumers and developers choose platforms under prices p_i ($p_i(x)$) and fees r_i as in Section S.1.

When μ is large and κ is small, the pair of ϕ and θ is in the parametric area in which personalized pricing benefits platforms and harms consumer welfare.

Reference

Anderson, Simon P., Bedre-Defolie, Özlem. 2025. App platform model. Mimeographed.

The fees for developers can be personalized when prices for consumers are personalized.
This setting aligns with Section 3.5 in Liu and Serfes (2013).
We compare two cases:
(i) Uniform pricing for both sides (observable prices and observable fees);
(ii) Private personalized pricing for both sides (private personalized prices and private personalized fees).

Uniform pricing for both sides

The prices for consumers and the fees for developers are observable as in Liu and Serfes (2013). Those prices influence the expectation for the masses of consumers belonging to the platforms, N1 and N2, then N1=N1e and N2=N2e. Those fees also influence the expectation for the masses of developers belonging to the platforms, D1 and D2, then D1=D1e and D2=D2e. Considering the four equations, we obtain the demand system in this case:

In[ ]:=
$$\text{Simplify}\left[\text{Solve}\left[\left\{n1 == \frac{t+\theta\,(d1-d2)-p1+p2}{2\,t},\ n2 == 1-\frac{t+\theta\,(d1-d2)-p1+p2}{2\,t},\ d1 == \phi\, n1 - l1,\ d2 == \phi\, n2 - l2\right\},\ \{n1, n2, d1, d2\}\right]\right]$$

Out[ ]=
$$\left\{\left\{n1 \to \frac{-p1+p2+t-l1\,\theta+l2\,\theta-\theta\,\phi}{2\,(t-\theta\,\phi)},\ n2 \to \frac{p1-p2+t+l1\,\theta-l2\,\theta-\theta\,\phi}{2\,t-2\,\theta\,\phi},\ d1 \to \frac{\phi\,(-p1+p2+t+l2\,\theta-\theta\,\phi)+l1\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ d2 \to \frac{\phi\,(p1-p2+t+l1\,\theta-\theta\,\phi)+l2\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)}\right\}\right\}$$

We set the demand system under uniform prices and fees:

In[1]:=
$$\text{demandu} = \left\{n1 \to \frac{-p1+p2+t-l1\,\theta+l2\,\theta-\theta\,\phi}{2\,(t-\theta\,\phi)},\ n2 \to \frac{p1-p2+t+l1\,\theta-l2\,\theta-\theta\,\phi}{2\,t-2\,\theta\,\phi},\ d1 \to \frac{\phi\,(-p1+p2+t+l2\,\theta-\theta\,\phi)+l1\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ d2 \to \frac{\phi\,(p1-p2+t+l1\,\theta-\theta\,\phi)+l2\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)}\right\};$$

We derive the FOCs:

In[2]:=
```
Simplify[D[p1 n1 + l1 d1 /. demandu, p1]]
Simplify[D[p1 n1 + l1 d1 /. demandu, l1]]
Simplify[D[p2 n2 + l2 d2 /. demandu, p2]]
Simplify[D[p2 n2 + l2 d2 /. demandu, l2]]
```

Out[2]= $$\frac{-2\,p1 + p2 + t - l1\,\theta + l2\,\theta - l1\,\phi - \theta\,\phi}{2\,(t - \theta\,\phi)}$$

Out[3]= $$\frac{-p1\,(\theta + \phi) + \phi\,(p2 + t + l2\,\theta - \theta\,\phi) + l1\,(-4\,t + 2\,\theta\,\phi)}{2\,(t - \theta\,\phi)}$$

Out[4]= $$\frac{p1 - 2\,p2 + t + l1\,\theta - l2\,\theta - l2\,\phi - \theta\,\phi}{2\,t - 2\,\theta\,\phi}$$

Out[5]= $$\frac{-p2\,(\theta + \phi) + \phi\,(p1 + t + l1\,\theta - \theta\,\phi) + l2\,(-4\,t + 2\,\theta\,\phi)}{2\,(t - \theta\,\phi)}$$

Solving the FOCs, we obtain the prices and fees:

In[ ]:= $$\text{Simplify}\Big[\text{Solve}\Big[\Big\{\frac{-2\,p1 + p2 + t - l1\,\theta + l2\,\theta - l1\,\phi - \theta\,\phi}{2\,(t - \theta\,\phi)} == 0,\ \frac{-p1\,(\theta + \phi) + \phi\,(p2 + t + l2\,\theta - \theta\,\phi) + l1\,(-4\,t + 2\,\theta\,\phi)}{2\,(t - \theta\,\phi)} == 0,\ \frac{p1 - 2\,p2 + t + l1\,\theta - l2\,\theta - l2\,\phi - \theta\,\phi}{2\,t - 2\,\theta\,\phi} == 0,\ \frac{-p2\,(\theta + \phi) + \phi\,(p1 + t + l1\,\theta - \theta\,\phi) + l2\,(-4\,t + 2\,\theta\,\phi)}{2\,(t - \theta\,\phi)} == 0\Big\}\ /.\ \text{demandu},\ \{p1,\ p2,\ l1,\ l2\}\Big]\Big]$$

Out[ ]= $$\Big\{\Big\{p1 \to t - \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ p2 \to t - \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ l1 \to \frac{1}{4}\,(-\theta + \phi),\ l2 \to \frac{1}{4}\,(-\theta + \phi)\Big\}\Big\}$$

We set prices and fees:

In[6]:= $$\text{priceu} = \Big\{p1 \to t - \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ p2 \to t - \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ l1 \to \frac{1}{4}\,(-\theta + \phi),\ l2 \to \frac{1}{4}\,(-\theta + \phi)\Big\};$$

This is the same as that under uniform pricing in the main model because the setting is the same.

We set the profit of each platform, the consumer surplus, the producer surplus, and the total surplus:

In[7]:=
```
profitu = Simplify[p1 n1 + l1 d1 /. demandu /. priceu]
```

Out[7]= $$\frac{1}{16}\,\left(8\,t - \theta^2 - 6\,\theta\,\phi - \phi^2\right)$$

In[8]:= $$\text{CSu} = \int_0^{n1} (w + \theta\,d1 - t\,x - p1)\,dx + \int_{n1}^{1} (w + \theta\,d2 - t\,(1 - x) - p2)\,dx\ /.\ \text{demandu}\ /.\ \text{priceu}\ //\ \text{Simplify}$$

Out[8]= $$\frac{1}{4}\,\left(-5\,t + 4\,w + \theta^2 + 4\,\theta\,\phi + \phi^2\right)$$

In[9]:= PSu = $\int_0^{d1} (\phi\, n1 - k - l1)\, dk + \int_0^{d2} (\phi\, n2 - k - l2)\, dk$ /. demandu /. priceu // Simplify

Out[9]= $\frac{1}{16}\ (\theta + \phi)^2$

In[10]:= TSu = 2 profitu + CSu + PSu // Simplify

Out[10]=

$\frac{1}{16}\ \left(-4\,t + 16\,w + 3\ (\theta + \phi)^2\right)$

To check the SOC, we set the profit of platform 1:

In[11]:= π1 = p1 n1 + l1 d1 /. demandu // Simplify

Out[11]=

$$\frac{-p1^2 + p1\ (p2 + t - l1\,\theta + l2\,\theta - l1\,\phi - \theta\,\phi) + l1\ (\phi\ (p2 + t + l2\,\theta - \theta\,\phi) + l1\ (-2\,t + \theta\,\phi))}{2\ (t - \theta\,\phi)}$$

The following is the SOC.

In[12]:= Reduce[D[D[π1, p1], p1] × D[D[π1, l1], l1] – D[D[π1, p1], l1]^2 > 0 && D[D[π1, p1], p1] < 0 &&
D[D[π1, l1], l1] < 0 && 0 < d1 ≤ 1 /. demandu /. priceu, t] // Simplify

Out[12]=

$\phi \in \mathbb{R}\ \&\&\ 0 < \theta + \phi \le 4\ \&\&\ 8\,t > \theta^2 + 6\,\theta\,\phi + \phi^2$

Personalized pricing for both sides

The prices for consumers and the fees for developers are *private*.
This setting aligns with that in Liu and Serfes (2013): personalized pricing does not benefit platforms.
We show the opposite result below.
The demand sizes depend on the expectation for the masses of consumers (N1e and N2e) and the masses of developers (D1e and D2e). Because platform i can induce developers with k to join in it iff ϕ Nie - k - l_i(k) ≥ 0, the indifferent developers are k = ϕ Nie, which equals Di. Considering the manner of the expectations, we derive the consumer demands and the developer demands:

In[∘]:= Simplify$\left[\text{Solve}\left[\left\{n1 == \frac{t + \theta\ (d1e - d2e)}{2\,t},\right.\right.\right.$
$\left.\left.\left. n2 == 1 - \frac{t + \theta\ (d1e - d2e)}{2\,t},\ d1 == \phi\, n1e,\ d2 == \phi\, n2e\right\},\ \{n1, n2, d1, d2\}\right]\right]$

Out[∘]=

$\left\{\left\{n1 \to \frac{t + d1e\,\theta - d2e\,\theta}{2\,t},\ n2 \to \frac{t - d1e\,\theta + d2e\,\theta}{2\,t},\ d1 \to n1e\,\phi,\ d2 \to n2e\,\phi\right\}\right\}$

We set the demands under the expectations:

In[13]:= demandp = $\left\{n1 \to \frac{t + d1e\,\theta - d2e\,\theta}{2\,t},\ n2 \to \frac{t - d1e\,\theta + d2e\,\theta}{2\,t},\ d1 \to n1e\,\phi,\ d2 \to n2e\,\phi\right\}$;

Under the expectations, the realized demand sizes match the expectations: N1 = N1e, N2 = N2e, D1 =D1e, and D2=D2e.
Considering the matching between the expectations and the realizations, we derive the

In[ ]:= **Simplify[Solve[{n1 == n1e, n2 == n2e, d1 == d1e, d2 == d2e} /. demandp, {n1e, n2e, d1e, d2e}]]**

Out[ ]=

$$\left\{\left\{n1e \to \frac{1}{2},\ n2e \to \frac{1}{2},\ d1e \to \frac{\phi}{2},\ d2e \to \frac{\phi}{2}\right\}\right\}$$

We set the expected demand sizes:

In[14]:= $$\textbf{demandep} = \left\{n1e \to \frac{1}{2},\ n2e \to \frac{1}{2},\ d1e \to \frac{\phi}{2},\ d2e \to \frac{\phi}{2}\right\};$$

Because the platforms employ personalized prices and fees, the profits depend on their price schedules for both sides:

In[15]:= $$\text{Simplify}\left[\int_0^{n1} (\theta\ (d1e - d2e) + t\ (1 - 2\,x))\, dx + \int_0^{d1} (\phi\, n1e - k)\, dk \text{ /. demandp /. demandep}\right]$$

$$\text{Simplify}\left[\int_{n1}^{1} (\theta\ (d2e - d1e) + t\ (2\,x - 1))\, dx + \int_0^{d2} (\phi\, n2e - k)\, dk \text{ /. demandp /. demandep}\right]$$

Out[15]=

$$\frac{1}{8}\left(2\,t + \phi^2\right)$$

Out[16]=

$$\frac{1}{8}\left(2\,t + \phi^2\right)$$

The price schedules are already embedded in the above integrals.
We set the profit of each platform:

In[17]:= **profitp =**

$$\text{Simplify}\left[\int_0^{n1} (\theta\ (d1e - d2e) + t\ (1 - 2\,x))\, dx + \int_0^{d1} (\phi\, n1e - k)\, dk \text{ /. demandp /. demandep}\right]$$

Out[17]=

$$\frac{1}{8}\left(2\,t + \phi^2\right)$$

We set the price schedules for consumers:

In[18]:= **p1x = (θ (d1e - d2e) + t (1 - 2 x)) /. demandp /. demandep;**
p2x = (θ (d2e - d1e) + t (2 x - 1)) /. demandp /. demandep;

We set the consumer surplus, the developers' surplus, and the total surplus:

In[20]:= $$\text{CSp} = \int_0^{n1} (w + \theta\, d1 - t\,x - p1x)\, dx + \int_{n1}^{1} (w + \theta\, d2 - t\,(1 - x) - p2x)\, dx \text{ /. demandp /. demandep //}$$
Simplify

Out[20]=

$$-\frac{3\,t}{4} + w + \frac{\theta\,\phi}{2}$$

In[21]:= $$\text{PSp} = \int_0^{d1} (\phi\, n1 - k - (\phi\, n1e - k))\, dk + \int_0^{d2} (\phi\, n2 - k - (\phi\, n1e - k))\, dk \text{ /. demandp /. demandep //}$$
Simplify

Out[21]=

0

In[22]:=
```
TSp = 2 profitp + CSp + PSp // Simplify
```
Out[22]=

$$-\frac{t}{4} + w + \frac{1}{4}\,\phi\,(2\,\theta + \phi)$$

Note that personalized fees allow platforms to completely exploit the developer surpluses. Therefore, PSp in the above is zero.

Comparison

The following is the second-order conditions:

$$8\,t > \theta^2 + 6\,\theta\,\phi + \phi^2,\ 0 < \theta + \phi \le 4$$

In[23]:=
```
Simplify[profitu - profitp]
Simplify[CSu - CSp]
Simplify[PSu - PSp]
Simplify[TSu - TSp]
```
Out[23]=

$$\frac{1}{16}\left(4\,t - \theta^2 - 6\,\theta\,\phi - 3\,\phi^2\right)$$

Out[24]=

$$\frac{1}{4}\left(-2\,t + (\theta + \phi)^2\right)$$

Out[25]=

$$\frac{1}{16}\,(\theta + \phi)^2$$

Out[26]=

$$\frac{1}{16}\left(3\,\theta^2 - 2\,\theta\,\phi - \phi^2\right)$$

We find that our main results for profits and consumer surplus hold even when platforms can offer personalized fees for developers.

The fees for developers are secret.

Uniform pricing

The prices for consumers are observable.
Those prices influence the expectation for the masses of consumers belonging to the platforms, N1 and N2, then N1=N1e and N2=N2e.
The expectation for the masses of developers are D1e and D2e.
We solve the following to derive the demand system:

In[1]:=
$$\text{Simplify}\left[\text{Solve}\left[\left\{n1 == \frac{t + \theta\ (d1e - d2e) - p1 + p2}{2\ t},\ n2 == 1 - \frac{t + \theta\ (d1e - d2e) - p1 + p2}{2\ t},\ d1 == \phi\ n1 - l1,\ d2 == \phi\ n2 - l2\right\},\ \{n1, n2, d1, d2\}\right]\right]$$

Out[1]=
$$\left\{\left\{n1 \to \frac{-p1 + p2 + t + d1e\ \theta - d2e\ \theta}{2\ t},\ n2 \to \frac{p1 - p2 + t - d1e\ \theta + d2e\ \theta}{2\ t},\ d1 \to -l1 + \frac{(-p1 + p2 + t + d1e\ \theta - d2e\ \theta)\ \phi}{2\ t},\ d2 \to -l2 + \frac{(p1 - p2 + t - d1e\ \theta + d2e\ \theta)\ \phi}{2\ t}\right\}\right\}$$

We set the demand system:

In[2]:=
$$\text{demandu} = \left\{n1 \to \frac{-p1 + p2 + t + d1e\ \theta - d2e\ \theta}{2\ t},\ n2 \to \frac{p1 - p2 + t - d1e\ \theta + d2e\ \theta}{2\ t},\ d1 \to -l1 + \frac{(-p1 + p2 + t + d1e\ \theta - d2e\ \theta)\ \phi}{2\ t},\ d2 \to -l2 + \frac{(p1 - p2 + t - d1e\ \theta + d2e\ \theta)\ \phi}{2\ t}\right\};$$

We derive the FOCs:

In[3]:=
```
Simplify[D[p1 n1 + l1 d1 /. demandu, p1]]
Simplify[D[p1 n1 + l1 d1 /. demandu, l1]]
Simplify[D[p2 n2 + l2 d2 /. demandu, p2]]
Simplify[D[p2 n2 + l2 d2 /. demandu, l2]]
```

Out[3]=
$$\frac{-2\ p1 + p2 + t + d1e\ \theta - d2e\ \theta - l1\ \phi}{2\ t}$$

Out[4]=
$$-2\ l1 + \frac{(-p1 + p2 + t + d1e\ \theta - d2e\ \theta)\ \phi}{2\ t}$$

Out[5]=
$$\frac{p1 - 2\ p2 + t - d1e\ \theta + d2e\ \theta - l2\ \phi}{2\ t}$$

Out[6]=
$$-2\ l2 + \frac{(p1 - p2 + t - d1e\ \theta + d2e\ \theta)\ \phi}{2\ t}$$

In this pricing stage, the expectation for the masses of developers belonging to the platforms coincides with the actual ones, D1=D1e and D2=D2e. We use those two equations and the four FOCs:

In[7]:=
```
Simplify[
 Solve[{(-2 p1 + p2 + t + d1e θ - d2e θ - l1 ϕ)/(2 t) == 0, -2 l1 + ((-p1 + p2 + t + d1e θ - d2e θ) ϕ)/(2 t) == 0,
   (p1 - 2 p2 + t - d1e θ + d2e θ - l2 ϕ)/(2 t) == 0, -2 l2 + ((p1 - p2 + t - d1e θ + d2e θ) ϕ)/(2 t) == 0,
   d1 == d1e, d2 == d2e} /. demandu, {p1, p2, l1, l2, d1e, d2e}]]
```

Out[7]= $\left\{\left\{p1 \to t - \frac{\phi^2}{4},\ p2 \to t - \frac{\phi^2}{4},\ l1 \to \frac{\phi}{4},\ l2 \to \frac{\phi}{4},\ d1e \to \frac{\phi}{4},\ d2e \to \frac{\phi}{4}\right\}\right\}$

Because the fees are secret, those fees cannot influence consumers' expectations.
Then, the interaction value of consumers disappears in this result.
We set the prices, the fees, and the expectations for the masses of developers:

In[8]:=
```
priceu = {p1 → t - ϕ^2/4, p2 → t - ϕ^2/4, l1 → ϕ/4, l2 → ϕ/4, d1e → ϕ/4, d2e → ϕ/4};
```

The above result is the same as that under uniform pricing in the main model with $\theta = 0$.

We set the profit of each platform, the consumer surplus, the developer surplus, and the total surplus:

In[9]:=
```
profitu = Simplify[p1 n1 + l1 d1 /. demandu /. priceu]
```

Out[9]= $\frac{1}{16}\left(8\,t - \phi^2\right)$

In[10]:=
```
CSu =
 ∫_0^n1 (w + θ d1 - t x - p1) dx + ∫_n1^1 (w + θ d2 - t (1 - x) - p2) dx /. demandu /. priceu // Simplify
```

Out[10]=

$$-\frac{5\,t}{4} + w + \frac{1}{4}\,\phi\,(\theta + \phi)$$

In[11]:=
```
PSu = ∫_0^d1 (ϕ n1 - k - l1) dk + ∫_0^d2 (ϕ n2 - k - l2) dk /. demandu /. priceu // Simplify
```

Out[11]=

$$\frac{\phi^2}{16}$$

In[12]:=
```
TSu = 2 profitu + CSu + PSu // Simplify
```

Out[12]=

$$-\frac{t}{4} + w + \frac{1}{16}\,\phi\,(4\,\theta + 3\,\phi)$$

We check the second-order condition. We set the profit function:

In[13]:=
```
π1 = p1 n1 + l1 d1 /. demandu // Simplify
```

Out[13]=

$$\frac{-p1^2 + p1\,(p2 + t + d1e\,\theta - d2e\,\theta - l1\,\phi) + l1\,(-2\,l1\,t + (p2 + t + d1e\,\theta - d2e\,\theta)\,\phi)}{2\,t}$$

The second-order condition is

In[14]:=
```
Reduce[D[D[π1, p1], p1] × D[D[π1, l1], l1] - D[D[π1, p1], l1]^2 > 0 && D[D[π1, p1], p1] < 0 &&
    D[D[π1, l1], l1] < 0 && 0 < d1 ≤ 1 /. demandu /. priceu, t] // Simplify
```

Out[14]=

$0 < \phi \leq 4 \,\&\&\, 8\,t > \phi^2$

Personalized pricing

Fees are secret and *uniform*.
The prices for consumers and the fees are private.
The demand system depends on the expectation for the masses of consumers (N1e and N2e) and the masses of developers (D1e and D2e).

In[ ]:=
```
Simplify[Solve[{n1 == (t + θ (d1e - d2e))/(2 t),
   n2 == 1 - (t + θ (d1e - d2e))/(2 t), d1 == ϕ n1e - l1, d2 == ϕ n2e - l2}, {n1, n2, d1, d2}]]
```

Out[ ]=

$$\left\{\left\{n1 \to \frac{t + d1e\,\theta - d2e\,\theta}{2\,t},\ n2 \to \frac{t - d1e\,\theta + d2e\,\theta}{2\,t},\ d1 \to -l1 + n1e\,\phi,\ d2 \to -l2 + n2e\,\phi\right\}\right\}$$

We set the demand system:

In[15]:=
```
demandp = {n1 → (t + d1e θ - d2e θ)/(2 t), n2 → (t - d1e θ + d2e θ)/(2 t), d1 → -l1 + n1e ϕ, d2 → -l2 + n2e ϕ};
```

Under the pricing schedules of personalized pricing, we derive the FOCs with respect to the fees:

In[16]:=
```
Simplify[D[∫_0^n1 (θ (d1e - d2e) + t (1 - 2 x)) dx + l1 d1 /. demandp, l1]]
Simplify[D[∫_n1^1 (θ (d2e - d1e) + t (2 x - 1)) dx + l2 d2 /. demandp, l2]]
```

Out[16]=

$-2\,l1 + n1e\,\phi$

Out[17]=

$-2\,l2 + n2e\,\phi$

In this pricing stage, the expectation for the masses of developers belonging to the platforms coincides with the actual ones, D1=D1e and D2=D2e. Also, N1=N1e and N2=N2e.

In[18]:=
```
Simplify[Solve[
  {-2 l1 + n1e ϕ == 0, -2 l2 + n2e ϕ == 0, n1 == n1e, n2 == n2e, d1 == d1e, d2 == d2e} /. demandp,
  {n1e, n2e, l1, l2, d1e, d2e}]]
```

Out[18]=

$$\left\{\left\{n1e \to \frac{1}{2},\ n2e \to \frac{1}{2},\ l1 \to \frac{\phi}{4},\ l2 \to \frac{\phi}{4},\ d1e \to \frac{\phi}{4},\ d2e \to \frac{\phi}{4}\right\}\right\}$$

In[19]:=
```
pricep = {n1e → 1/2, n2e → 1/2, l1 → ϕ/4, l2 → ϕ/4, d1e → ϕ/4, d2e → ϕ/4};
```

Because the fees do not influence the expectations of consumers, the interaction value of each consumer θ disappears.
However, platforms correctly expect the masses of consumers belonging to them, and they charge the fees that depend on the interaction value of each developer ϕ. The above result is the same as

that under personalized pricing in the main model with $\theta = 0$.

The profits of each platform:

In[20]:=
```
profitp = Simplify[∫_0^n1 (θ (d1e - d2e) + t (1 - 2 x)) ⅆx + l1 d1 /. demandp /. pricep]
```

Out[20]=

$$\frac{1}{16}\left(4\,t + \phi^2\right)$$

The price schedules

In[21]:=
```
p1x = (θ (d1e - d2e) + t (1 - 2 x)) /. demandp /. pricep;
p2x = (θ (d2e - d1e) + t (2 x - 1)) /. demandp /. pricep;
```

The consumer surplus is

In[23]:=
```
CSp =
  ∫_0^n1 (w + θ d1 - t x - p1x) ⅆx + ∫_n1^1 (w + θ d2 - t (1 - x) - p2x) ⅆx /. demandp /. pricep // Simplify
```

Out[23]=

$$-\frac{3\,t}{4} + w + \frac{\theta\,\phi}{4}$$

The developer surplus is

In[24]:=
```
PSp = ∫_0^d1 (ϕ n1 - k - l1) ⅆk + ∫_0^d2 (ϕ n2 - k - l2) ⅆk /. demandp /. pricep // Simplify
```

Out[24]=

$$\frac{\phi^2}{16}$$

The total surplus is

In[25]:=
```
TSp = 2 profitp + CSp + PSp // Simplify
```

Out[25]=

$$-\frac{t}{4} + w + \frac{1}{16}\,\phi\,(4\,\theta + 3\,\phi)$$

We check the second-order condition. We set the profit function:

In[26]:=
```
π2 = ∫_0^n1 (θ (d1e - d2e) + t (1 - 2 x)) ⅆx + l1 d1 /. demandp // Simplify
```

Out[26]=

$$-l1^2 + \frac{(t + (d1e - d2e)\,\theta)^2}{4\,t} + l1\,n1e\,\phi$$

The second-order condition is

In[27]:=
```
Reduce[D[D[π2, l1], l1] < 0 && 0 < d1 ≤ 1 /. demandp /. pricep, t] // Simplify
```

Out[27]=

$$0 < \phi \leq 4$$

Comparison

We compare the profits, the consumer surplus, the developer surplus, and the total surplus.

In[28]:=
```
Simplify[profitu - profitp]
Simplify[CSu - CSp]
Simplify[PSu - PSp]
Simplify[TSu - TSp]
```

Out[28]=

$$\frac{1}{8}\left(2\,t-\phi^2\right)$$

Out[29]=

$$\frac{1}{4}\left(-2\,t+\phi^2\right)$$

Out[30]=

0

Out[31]=

0

The comparison for the profits and the consumer surplus is qualitatively similar to that in the main model.

We assume that platforms offer observable personalized prices.
In this file, we omit the calculation of uniform pricing.

Personalized pricing

Following the mathematical procedure in Liu and Serfes (2008), a working paper version of Liu and Serfes (2013), we consider the following price schedule:
Platform 1 adds a subsidy b1 to its price schedule p1(x), and platform 2 adds a subsidy b2 to its price schedule p2(x).

Given the expectations for the masses of developers in the platforms, D1e and D2e,
the amount of consumers in platform 1 is $\mathbf{N1} = \frac{b1-b2+t+D1e\,\theta-D2e\,\theta}{2\,t}$ and that in platform 2 is $\mathbf{N2} = 1 - \frac{b1-b2+t+D1e\,\theta-D2e\,\theta}{2\,t}$.
Considering this fact, we derive the demand system:

In[1]:=
$$\text{Simplify}\Big[\text{Solve}\Big[\Big\{\phi\left(\frac{b1-b2+t+D1e\,\theta-D2e\,\theta}{2\,t}\right)-l1==D1,\ \phi\left(1-\frac{b1-b2+t+D1e\,\theta-D2e\,\theta}{2\,t}\right)-l2==D2,\ N1==\frac{b1-b2+t+D1e\,\theta-D2e\,\theta}{2\,t},\ N2==1-\frac{b1-b2+t+D1e\,\theta-D2e\,\theta}{2\,t},\ D1e==D1,\ D2e==D2\Big\},\ \{D1e,D2e,D1,D2,N1,N2\}\Big]\Big]$$

Out[1]=
$$\Big\{\Big\{D1e\to\frac{\phi\,(b1-b2+t+l2\,\theta-\theta\,\phi)+l1\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ D2e\to\frac{\phi\,(-b1+b2+t+l1\,\theta-\theta\,\phi)+l2\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ D1\to\frac{\phi\,(b1-b2+t+l2\,\theta-\theta\,\phi)+l1\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ D2\to\frac{\phi\,(-b1+b2+t+l1\,\theta-\theta\,\phi)+l2\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ N1\to\frac{b1-b2+t-l1\,\theta+l2\,\theta-\theta\,\phi}{2\,t-2\,\theta\,\phi},\ N2\to\frac{-b1+b2+t+l1\,\theta-l2\,\theta-\theta\,\phi}{2\,(t-\theta\,\phi)}\Big\}\Big\}$$

Using the above result, we check the utility difference at the indifferent consumers is zero (p1(x) = p2(x) = 0 at the indifferent consumers):

In[ ]:=
$$\text{Simplify}\Big[D1e\,\theta-0+b1-t\,x-(D2e\,\theta-0+b2-t\,(1-x))\ /.\ \Big\{D1e\to\frac{\phi\,(b1-b2+t+l2\,\theta-\theta\,\phi)+l1\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)},\ D2e\to\frac{\phi\,(-b1+b2+t+l1\,\theta-\theta\,\phi)+l2\,(-2\,t+\theta\,\phi)}{2\,(t-\theta\,\phi)}\Big\}\ /.\ x\to\frac{b1-b2+t-l1\,\theta+l2\,\theta-\theta\,\phi}{2\,t-2\,\theta\,\phi}\Big]$$

Out[ ]=

0

We derive the price schedules except for the additional discount ϵ (bi in this file) mentioned in Liu and Serfes (Working paper version) on page 10, p1(x) and p2(x):

In[2]:=
```
Solve[D1e θ - (p1x - b1) - t x == D2e θ - (0 - b2) - t (1 - x) /.
  {D1e -> (ϕ (b1 - b2 + t + l2 θ - θ ϕ) + l1 (-2 t + θ ϕ))/(2 (t - θ ϕ)),
   D2e -> (ϕ (-b1 + b2 + t + l1 θ - θ ϕ) + l2 (-2 t + θ ϕ))/(2 (t - θ ϕ))}, p1x]
Solve[D1e θ - (0 - b1) - t x == D2e θ - (p2x - b2) - t (1 - x) /.
  {D1e -> (ϕ (b1 - b2 + t + l2 θ - θ ϕ) + l1 (-2 t + θ ϕ))/(2 (t - θ ϕ)),
   D2e -> (ϕ (-b1 + b2 + t + l1 θ - θ ϕ) + l2 (-2 t + θ ϕ))/(2 (t - θ ϕ))}, p2x]
```

Out[2]= $\left\{\left\{p1x \to \frac{b1\,t - b2\,t + t^2 - 2\,t^2\,x - l1\,t\,\theta + l2\,t\,\theta - t\,\theta\,\phi + 2\,t\,x\,\theta\,\phi}{t - \theta\,\phi}\right\}\right\}$

Out[3]= $\left\{\left\{p2x \to \frac{-b1\,t + b2\,t - t^2 + 2\,t^2\,x + l1\,t\,\theta - l2\,t\,\theta + t\,\theta\,\phi - 2\,t\,x\,\theta\,\phi}{t - \theta\,\phi}\right\}\right\}$

We can simplify p1x and p2x as in each of the first term:

In[4]:=
```
Simplify[(t (1 - 2 x) + (t (b1 - b2 + (l2 - l1) θ))/(t - θ ϕ)) -
  (b1 t - b2 t + t^2 - 2 t^2 x - l1 t θ + l2 t θ - t θ ϕ + 2 t x θ ϕ)/(t - θ ϕ)]
Simplify[(t (2 x - 1) + (t (b2 - b1 + (l1 - l2) θ))/(t - θ ϕ)) -
  (-b1 t + b2 t - t^2 + 2 t^2 x + l1 t θ - l2 t θ + t θ ϕ - 2 t x θ ϕ)/(t - θ ϕ)]
```

Out[4]= 0

Out[5]= 0

Note that, if b1 and b2 were 0, the above p1x and p2x would be the same as those in the price schedules under secret personalized pricing.

The prices are zero at the location of the indifferent consumers:

In[6]:=
```
Simplify[t (1 - 2 x) + (t (b1 - b2 + (l2 - l1) θ))/(t - θ ϕ) /. x -> (b1 - b2 + t - l1 θ + l2 θ - θ ϕ)/(2 t - 2 θ ϕ)]
Simplify[t (2 x - 1) + (t (b2 - b1 + (l1 - l2) θ))/(t - θ ϕ) /. x -> (b1 - b2 + t - l1 θ + l2 θ - θ ϕ)/(2 t - 2 θ ϕ)]
```

Out[6]= 0

Out[7]= 0

We set the profit functions:

In[8]:= $\pi 1 = \int_0^{N1} \left(t\,(1 - 2\,x) + \frac{t\,(b1 - b2 + (l2 - l1)\,\theta)}{t - \theta\,\phi} - b1 \right) dx + l1$
$\left(\frac{\phi}{2} + \frac{\phi\,(b1 - b2) + \theta\,\phi\,l2 - (2\,t - \theta\,\phi)\,l1}{2\,(t - \theta\,\phi)} \right)$ /. $N1 \to \frac{b1 - b2 + t - l1\,\theta + l2\,\theta - \theta\,\phi}{2\,t - 2\,\theta\,\phi}$ // Simplify

$\pi 2 = \int_{N1}^{1} \left(t\,(2\,x - 1) + \frac{t\,(b2 - b1 + (l1 - l2)\,\theta)}{t - \theta\,\phi} - b2 \right) dx + l2$
$\left(\frac{\phi}{2} + \frac{\phi\,(b2 - b1) + \theta\,\phi\,l1 - (2\,t - \theta\,\phi)\,l2}{2\,(t - \theta\,\phi)} \right)$ /. $N1 \to \frac{b1 - b2 + t - l1\,\theta + l2\,\theta - \theta\,\phi}{2\,t - 2\,\theta\,\phi}$ // Simplify

Out[8]= $\frac{1}{4\,(t - \theta\,\phi)^2}$
$\big(b2^2\,t - 4\,l1^2\,t^2 + t^3 - 2\,l1\,t^2\,\theta + 2\,l2\,t^2\,\theta + l1^2\,t\,\theta^2 - 2\,l1\,l2\,t\,\theta^2 + l2^2\,t\,\theta^2 + 2\,l1\,t^2\,\phi + 6\,l1^2\,t\,\theta\,\phi +$
$2\,l1\,l2\,t\,\theta\,\phi - 2\,t^2\,\theta\,\phi + 2\,l1\,t\,\theta^2\,\phi - 2\,l2\,t\,\theta^2\,\phi - 4\,l1\,t\,\theta\,\phi^2 - 2\,l1^2\,\theta^2\,\phi^2 - 2\,l1\,l2\,\theta^2\,\phi^2 +$
$t\,\theta^2\,\phi^2 + 2\,l1\,\theta^2\,\phi^3 - b1^2\,(t - 2\,\theta\,\phi) - 2\,b2\,\big(t^2 + t\,\theta\,(l2 - \phi) - l1\,\theta\,\phi^2 + l1\,t\,(-\theta + \phi)\big) -$
$2\,b1\,\phi\,(-l1\,t + l1\,\theta\,(\theta + \phi) + \theta\,(b2 - t + \theta\,(-l2 + \phi)))\big)$

Out[9]= $\frac{1}{4\,(t - \theta\,\phi)^2}$
$\big(b1^2\,t - 4\,l2^2\,t^2 + t^3 + 2\,l1\,t^2\,\theta - 2\,l2\,t^2\,\theta + l1^2\,t\,\theta^2 - 2\,l1\,l2\,t\,\theta^2 + l2^2\,t\,\theta^2 + 2\,l2\,t^2\,\phi + 2\,l1\,l2\,t\,\theta\,\phi +$
$6\,l2^2\,t\,\theta\,\phi - 2\,t^2\,\theta\,\phi - 2\,l1\,t\,\theta^2\,\phi + 2\,l2\,t\,\theta^2\,\phi - 4\,l2\,t\,\theta\,\phi^2 - 2\,l1\,l2\,\theta^2\,\phi^2 - 2\,l2^2\,\theta^2\,\phi^2 +$
$t\,\theta^2\,\phi^2 + 2\,l2\,\theta^2\,\phi^3 - b2^2\,(t - 2\,\theta\,\phi) - 2\,b1\,\big(t^2 + \theta\,\phi\,(b2 - l2\,\phi) + t\,(l1\,\theta - l2\,\theta + l2\,\phi - \theta\,\phi)\big) +$
$2\,b2\,\phi\,(\theta\,(t + \theta\,(l1 - \phi)) + l2\,(t - \theta\,(\theta + \phi)))\big)$

Solving the first-order conditions, we have the levels of the “subsidies” for consumers and the fees for developers:

In[10]:= `Solve[D[π1, l1] == 0 && D[π1, b1] == 0 && D[π2, l2] == 0 && D[π2, b2] == 0, {l1, b1, l2, b2}] // Simplify`

Out[10]=
$\left\{\left\{l1 \to \frac{1}{4}\,(-\theta + \phi),\ b1 \to \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ l2 \to \frac{1}{4}\,(-\theta + \phi),\ b2 \to \frac{1}{4}\,\phi\,(3\,\theta + \phi)\right\}\right\}$

The price schedules of the firms are

In[11]:= $\text{Simplify}\Big[t\,(1 - 2\,x) + \frac{t\,(b1 - b2 + (l2 - l1)\,\theta)}{t - \theta\,\phi} - b1$ /.
$\left\{l1 \to \frac{1}{4}\,(-\theta + \phi),\ b1 \to \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ l2 \to \frac{1}{4}\,(-\theta + \phi),\ b2 \to \frac{1}{4}\,\phi\,(3\,\theta + \phi)\right\}\Big]$

$\text{Simplify}\Big[t\,(2\,x - 1) + \frac{t\,(b2 - b1 + (l1 - l2)\,\theta)}{t - \theta\,\phi} - b2$ /.
$\left\{l1 \to \frac{1}{4}\,(-\theta + \phi),\ b1 \to \frac{1}{4}\,\phi\,(3\,\theta + \phi),\ l2 \to \frac{1}{4}\,(-\theta + \phi),\ b2 \to \frac{1}{4}\,\phi\,(3\,\theta + \phi)\right\}\Big]$

Out[11]=
$t - 2\,t\,x - \frac{1}{4}\,\phi\,(3\,\theta + \phi)$

Out[12]=
$\frac{1}{4}\,(t\,(-4 + 8\,x) - \phi\,(3\,\theta + \phi))$

The above prices for consumer x are lower than those in private personalized pricing.

We derive the second-order conditions:

In[13]:=
```
Reduce[
 D[D[π1, l1], l1] × D[D[π1, b1], b1] - D[D[π1, l1], b1]^2 > 0 && t > 0 && θ > 0 && ϕ > 0, t]
```

Out[13]=

$$\phi > 0 \&\& \theta > 0 \&\& t > \frac{1}{4}\left(\theta^2 + 6\,\theta\,\phi + \phi^2\right)$$

The equilibrium profits are

In[14]:=
```
π1 /. {l1 → 1/4 (-θ + ϕ), b1 → 1/4 ϕ (3 θ + ϕ), l2 → 1/4 (-θ + ϕ), b2 → 1/4 ϕ (3 θ + ϕ)} // Simplify
π2 /. {l1 → 1/4 (-θ + ϕ), b1 → 1/4 ϕ (3 θ + ϕ), l2 → 1/4 (-θ + ϕ), b2 → 1/4 ϕ (3 θ + ϕ)} // Simplify
```

Out[14]=

$$\frac{1}{16}\left(4\,t - \theta^2 - 6\,\theta\,\phi - \phi^2\right)$$

Out[15]=

$$\frac{1}{16}\left(4\,t - \theta^2 - 6\,\theta\,\phi - \phi^2\right)$$

From the above result, we find that the above profits are lower than those under uniform pricing (see Lemma 1 in our paper).

We set the derived l1, l2, b1, and b2:

In[16]:=
```
lbo = {l1 → 1/4 (-θ + ϕ), b1 → 1/4 ϕ (3 θ + ϕ), l2 → 1/4 (-θ + ϕ), b2 → 1/4 ϕ (3 θ + ϕ)}
```

Out[16]=

$$\left\{l1 \to \frac{1}{4}(-\theta + \phi),\ b1 \to \frac{1}{4}\phi\,(3\,\theta + \phi),\ l2 \to \frac{1}{4}(-\theta + \phi),\ b2 \to \frac{1}{4}\phi\,(3\,\theta + \phi)\right\}$$

In[ ]:=
```
Reduce[0 < (ϕ (b1 - b2 + t + l2 θ - θ ϕ) + l1 (-2 t + θ ϕ))/(2 (t - θ ϕ)) < 1 && θ > 0 && ϕ > 0 /. lbo]
```

Out[ ]=

$$t \in \mathbb{R} \&\& 0 < \phi < 4 \&\& 0 < \theta < 4 - \phi$$

In[ ]:=
```
Reduce[1/16 (4 t - θ^2 - 6 θ ϕ - ϕ^2) > 0 && t > 0 && θ > 0 && ϕ > 0]
```

Out[ ]=

$$\phi > 0 \&\& \theta > 0 \&\& t > \frac{1}{4}\left(\theta^2 + 6\,\theta\,\phi + \phi^2\right)$$

We set the price schedules p1(x) and p2(x) (that do not include b1 and b2) and the demands:

In[17]:=
$$px = \left\{p1x \to \frac{b1\,t - b2\,t + t^2 - 2\,t^2\,x - l1\,t\,\theta + l2\,t\,\theta - t\,\theta\,\phi + 2\,t\,x\,\theta\,\phi}{t - \theta\,\phi},\ p2x \to \frac{-b1\,t + b2\,t - t^2 + 2\,t^2\,x + l1\,t\,\theta - l2\,t\,\theta + t\,\theta\,\phi - 2\,t\,x\,\theta\,\phi}{t - \theta\,\phi}\right\} \text{ /. lbo // Simplify}$$

$$De = \left\{D1e \to \frac{\phi\,(b1 - b2 + t + l2\,\theta - \theta\,\phi) + l1\,(-2\,t + \theta\,\phi)}{2\,(t - \theta\,\phi)},\ D2e \to \frac{\phi\,(-b1 + b2 + t + l1\,\theta - \theta\,\phi) + l2\,(-2\,t + \theta\,\phi)}{2\,(t - \theta\,\phi)},\ D1 \to \frac{\phi\,(b1 - b2 + t + l2\,\theta - \theta\,\phi) + l1\,(-2\,t + \theta\,\phi)}{2\,(t - \theta\,\phi)},\ D2 \to \frac{\phi\,(-b1 + b2 + t + l1\,\theta - \theta\,\phi) + l2\,(-2\,t + \theta\,\phi)}{2\,(t - \theta\,\phi)},\ N1 \to \frac{b1 - b2 + t - l1\,\theta + l2\,\theta - \theta\,\phi}{2\,t - 2\,\theta\,\phi},\ N2 \to \frac{-b1 + b2 + t + l1\,\theta - l2\,\theta - \theta\,\phi}{2\,(t - \theta\,\phi)}\right\} \text{ /. lbo // Simplify}$$

Out[17]=
$$\{p1x \to t - 2\,t\,x,\ p2x \to t\,(-1 + 2\,x)\}$$

Out[18]=
$$\left\{D1e \to \frac{\theta + \phi}{4},\ D2e \to \frac{\theta + \phi}{4},\ D1 \to \frac{\theta + \phi}{4},\ D2 \to \frac{\theta + \phi}{4},\ N1 \to \frac{1}{2},\ N2 \to \frac{1}{2}\right\}$$

In[19]:=
$$CS = \int_0^{N1} (w + D1e\,\theta - p1x + b1 - t\,x)\,dx + \int_{N1}^{1} (w + D2e\,\theta - p2x + b2 - t\,(1 - x))\,dx \text{ /. De /. px /. lbo // Simplify}$$

Out[19]=
$$\frac{1}{4}\left(-t + 4\,w + \theta^2 + 4\,\theta\,\phi + \phi^2\right)$$

$$w - \frac{t}{4} + \frac{\theta^2 + 4\,\theta\,\phi + \phi^2}{4}$$

In[20]:=
$$PS = \int_0^{D1} (\phi\,N1 - k - l1)\,dk + \int_0^{D2} (\phi\,N2 - k - l2)\,dk \text{ /. De /. px /. lbo // Simplify}$$

Out[20]=
$$\frac{1}{16}\,(\theta + \phi)^2$$

Comparison

The result under observable personalized pricing

In[21]:= $\pi 1o = \frac{1}{16}\left(4\,t - \theta^2 - 6\,\theta\,\phi - \phi^2\right);$

$CSo = \frac{1}{4}\left(-t + 4\,w + \theta^2 + 4\,\theta\,\phi + \phi^2\right);$

$PSo = \frac{1}{16}\,(\theta + \phi)^2;$

$D1o = \frac{\theta + \phi}{4};$

The result under private personalized pricing

In[25]:= $\pi 1p = \frac{\left(4\,t + \phi^2\right)\left(4\,t^2 + 2\,\theta^2\,\phi^2 - t\,\theta\,(\theta + 6\,\phi)\right)}{4\,(4\,t - 3\,\theta\,\phi)^2};$

$CSp = \frac{-12\,t^2 + 16\,t\,w + t\,\theta\,(4\,\theta + 13\,\phi) - 2\,\theta\,\phi\,(6\,w + \theta\,\phi)}{4\,(4\,t - 3\,\theta\,\phi)};$

$PSp = \frac{\left(\theta\,\phi^2 - 2\,t\,(\theta + \phi)\right)^2}{4\,(4\,t - 3\,\theta\,\phi)^2};$

$D1p = \frac{-\theta\,\phi^2 + 2\,t\,(\theta + \phi)}{8\,t - 6\,\theta\,\phi};$

The result under uniform pricing

In[29]:= $\pi 1u = \frac{1}{16}\left(8\,t - \theta^2 - 6\,\theta\,\phi - \phi^2\right);$

$CSu = \frac{1}{4}\left(-5\,t + 4\,w + \theta^2 + 4\,\theta\,\phi + \phi^2\right);$

$PSu = \frac{1}{16}\,(\theta + \phi)^2;$

$D1u = \frac{\theta + \phi}{4};$

We set several parametric restrictions (the SOC and so on).

In[33]:= $Condo = Reduce\left[\left(\phi > 0\ \&\&\ \theta > 0\ \&\&\ t > \frac{1}{4}\left(\theta^2 + 6\,\theta\,\phi + \phi^2\right)\right)\ \&\&\ 0 < \phi < 4\ \&\&\ 0 < \theta < 4 - \phi\right]\ //\ Simplify$

Out[33]=

$0 < \theta < 4\ \&\&\ \phi > 0\ \&\&\ \theta + \phi < 4\ \&\&\ 4\,t > \theta^2 + 6\,\theta\,\phi + \phi^2$

In[34]:= $Condp = Reduce\left[t > \frac{1}{8}\left(\theta^2 + 6\,\theta\,\phi\right) + \frac{1}{8}\sqrt{\theta^4 + 12\,\theta^3\,\phi + 4\,\theta^2\,\phi^2}\ \&\&\ \theta > 0\ \&\&\ \phi > 0\ \&\&\ 0 < \frac{-\theta\,\phi^2 + 2\,t\,(\theta + \phi)}{8\,t - 6\,\theta\,\phi} < 1\right]\ //\ Simplify$

Out[34]=

$\left(t > \frac{\theta\,(-6 + \phi)\,\phi}{2\,(-4 + \theta + \phi)}\ \&\&\ \theta + \phi < 4\ \&\&\ ((\theta > 0\ \&\&\ 2 < \phi < 4)\ ||\ (\phi > 0\ \&\&\ 4\,\phi + \theta\,(2 + \phi) > 8\ \&\&\ \phi \le 2))\right)\ ||$

$\left(0 < \theta \le \frac{8 - 4\,\phi}{2 + \phi}\ \&\&\ 0 < \phi \le 2\ \&\&\ 8\,t > \theta^2 + 6\,\theta\,\phi + \sqrt{\theta^2\left(\theta^2 + 12\,\theta\,\phi + 4\,\phi^2\right)}\right)$

In[35]:= **Condu = Reduce$\left[t > \frac{1}{8}\left(\theta^2 + 6\,\theta\,\phi + \phi^2\right)\ \&\&\ 0 < \theta + \phi < 4\right]$ // Simplify**

Out[35]=

$\phi \in \mathbb{R}\ \&\&\ 0 < \theta + \phi < 4\ \&\&\ 8\,t > \theta^2 + 6\,\theta\,\phi + \phi^2$

In[36]:= **Reduce[Condo && Condp] // Simplify**

Out[36]=

$$0 < \theta < 4\ \&\&\ \left(\left(0 < \phi \le \text{Root}\left[-4\,\theta^2 + \theta^3 + \left(-12\,\theta + 7\,\theta^2\right)\#1 + (-4 + 5\,\theta)\,\#1^2 + \#1^3\ \&,\ 3\right]\ \&\&\ 4\,t > \theta^2 + 6\,\theta\,\phi + \phi^2\right)\ ||\ \left(\phi > \text{Root}\left[-4\,\theta^2 + \theta^3 + \left(-12\,\theta + 7\,\theta^2\right)\#1 + (-4 + 5\,\theta)\,\#1^2 + \#1^3\ \&,\ 3\right]\ \&\&\ \theta + \phi < 4\ \&\&\ t > \frac{\theta\,(-6 + \phi)\,\phi}{2\,(-4 + \theta + \phi)}\right)\right)$$

In[37]:= **Reduce[Condo && Condu] // Simplify**

Out[37]=

$0 < \phi < 4\ \&\&\ \theta > 0\ \&\&\ \theta + \phi < 4\ \&\&\ 4\,t > \theta^2 + 6\,\theta\,\phi + \phi^2$

observable vs unobservable

In[38]:= **Reduce[π1o > π1p && Condo && Condp] // Simplify**

Out[38]=

False

In[39]:= **Reduce[D1o > D1p && Condo && Condp] // Simplify**

Out[39]=

False

In[40]:= **Reduce[CSo < CSp && Condo && Condp] // Simplify**

Out[40]=

False

In[41]:= **Reduce[PSo > PSp && Condo && Condp] // Simplify**

Out[41]=

False

observable vs uniform

In[42]:= **Reduce[π1o > π1u && Condo && Condu] // Simplify**

Out[42]=

False

In[43]:= **Reduce[D1o ≠ D1u && Condo && Condu] // Simplify**

Out[43]=

False

In[44]:= **Reduce[CSo < CSu && Condo && Condu] // Simplify**

Out[44]=

False

In[45]:= **Reduce[PSo ≠ PSu && Condo && Condu] // Simplify**

Out[45]=

False